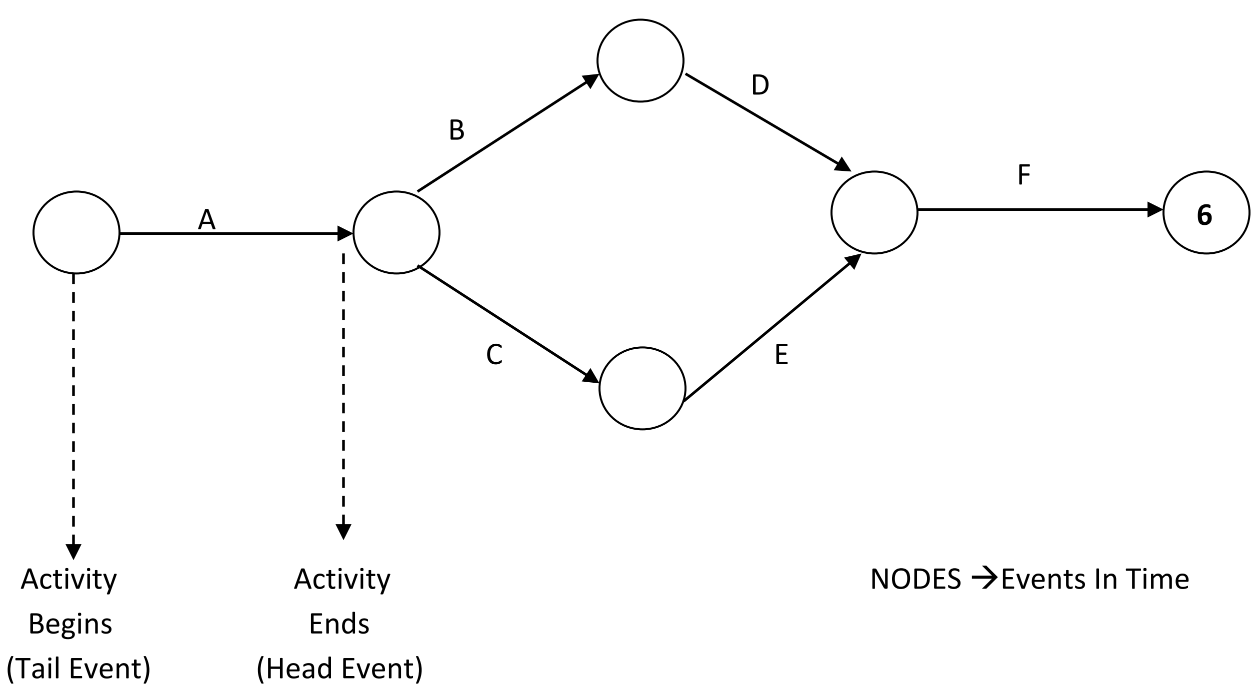
Project Management Techniques (PMT)

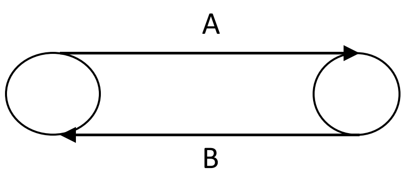
- Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT) are 2 main basic techniques used in project management.
- Example:
    - Construction of a house.
    - Construction of a bridge.
    - Construction of a power plant.
    - Large industrial projects etc.
    - Organizing research and development activities etc.

ACTIVITY	DESCRIPTION	DURATION (WEEKS)	IMMEDIATE PREDECESSORS
A	Obtain budget approval	2	-
B	Obtain machine	5	A
C	Hire operator	1	A
D	Install machine	1	B
E	Train operator	6	C
F	Produce the goods	1	D, E

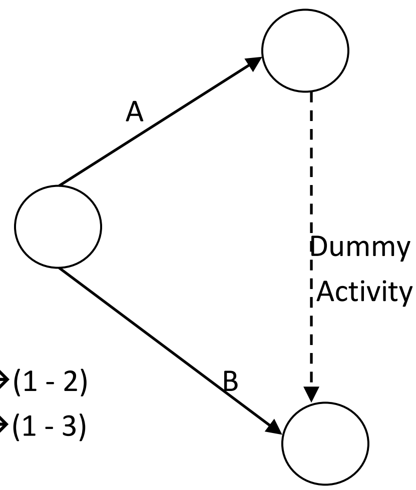


RULES OF NETORK CONSTRUCTION:

1. Each defined activity is represented by one and only one arrow in the network.
2. Before an activity begins, all activities preceding it must be completed.
3. The arrow direction indicates general progression in time. The arrow head represents the point in time at which the activity completion event takes place.
4. Events are identified by numbers. There should be no duplication of numbers.
5. A network should have only one initial and one terminal node.
6. Parallel activities between two events, without intervening events, are prohibited i.e., 2 activities cannot have same head and tail events.



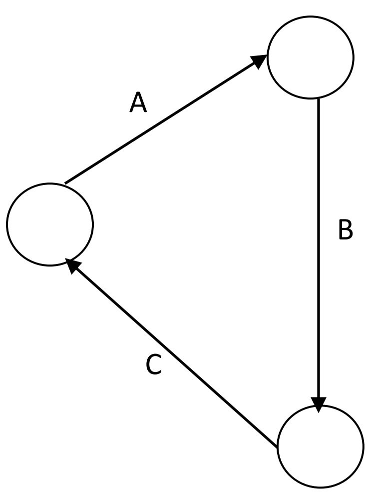
A → (1 - 2)
B → (1 - 2)



A →(1 - 2)
B →(1 - 3)

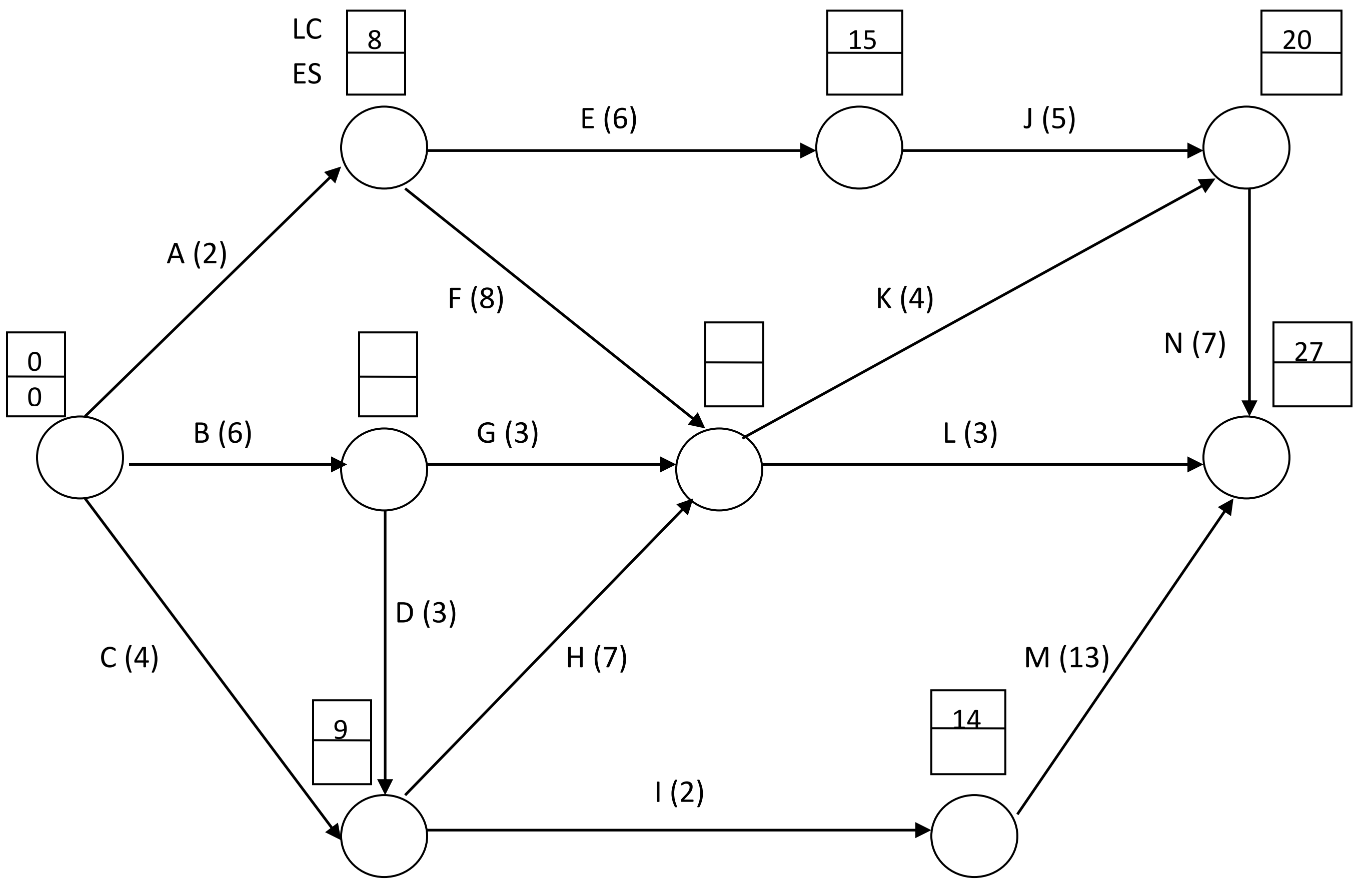
– Minimum number of dummy activities should be used for efficient network.

7. Looping is not permitted in a network.



→ If A precedes B and B precedes C, then C cannot precede A

Critical Path Method (CPM)



PHASE 1: Earliest Start Times (ES) {Forward Pass}

Let D_{ij} → duration of activity (i, j).

ES_j → Earliest start times of all the activities at node j.

$ES_j = \max (ES_i + D_{ij})$

Node 1: $ES_1 = 0$

Node 2: $ES_2 = ES_1 + D_{12} = 0 + 2 = 2$

Node 3: $ES_3 = ES_1 + D_{13} = 0 + 6 = 6$

Node 4: $ES_4 = \max (ES_i + D_{i4})$ (i = 1, 3)

$= \max [(ES_1 + D_{14}), (ES_3 + D_{34})]$

$= \max [4, (6 + 3)] = 9$

Node 5: $ES_5 = ES_2 + D_{25} = 2 + 6 = 8$

Node 6: $ES_6 = \max (ES_i + D_{i6})$ (i = 2, 3, 4)

$= 16$

Node 7: $ES_7 = ES_4 + D_{47} = 9 + 2 = 11$

Node 8: $ES_8 = \max (ES_i + D_{i8})$ (i = 5, 6)

$= \max [(ES_5 + D_{58}), (ES_6 + D_{68})]$

$= \max [(8 + 5), (16 + 4)] = 20$

Node 9: $ES_9 = \max (ES_i + D_{i8})$ (i = 6, 7, 8)

$= \max [(ES_6 + D_{69}), (ES_7 + D_{79}), (ES_8 + D_{89})]$

$= \max [(16 + 3), (11 + 13), (20 + 7)]$

$= \max [19, 24, 27] = 27$

PHASE 2: Latest Completion Time (LC_j) {Backward Pass}

Let $LC_j \rightarrow$ latest completion time of all activities at node j.

$LC_i = \min (LC_j - D_{ij})$

Node 9: $LC_9 = ES_9 = 27$

Node 8: $LC_8 = LC_9 - D_{89} = 27 - 7 = 20$

Node 7: $LC_7 = LC_9 - D_{79} = 27 - 13 = 14$

Node 6: $LC_6 = \min (LC_j - D_{6j}) = 16$

Node 5: $LC_5 = 15$

Node 4: $LC_4 = 9$

Node 3: $LC_3 = 6$

Node 2: $LC_2 = 8$

Node 1: $LC_1 = 0$

i	ES_i	LC_i
1	0	0
2	2	8
3	6	6
4	9	9
5	8	15
6	16	16
7	11	14
8	20	20
9	27	27

Critical Activities: B → D → H → K → N.

Project Completion Time: 6 + 3 + 7 + 4 +7 = 27 months.

FLOATS

Total FLOATS: It is the amount of time that the activity completion time can be delayed without affecting the *project completion time*.

$$TF_{ij} = LC_j - ES_i - D_{ij}$$

Free FLOATS: It is the amount of time that the activity completion time can be delayed without affecting the earliest start time of the immediate successor activities in the network.

$$FF_{ij} = ES_j - ES_i - D_{ij}$$

Activity (i, j)	Duration D_{ij}	Total FLOATS TF_{ij}	Free FLOATS FF_{ij}
1 - 2	2	6	0
1 – 3	6	0	0
1 - 4	4	5	5
2 - 5	6	7	0
2 - 6	8	6	6
3 - 4	3	0	0
3 – 6	3	7	7
4 – 6	7	0	0
4 - 7	2	3	0
5 – 8	5	7	7
6 – 8	4	0	0
6 – 9	3	8	8
7 – 9	13	3	3
8 - 9	7	0	0

Any critical activity will have zero Total FLOAT and zero Free FLOAT.

PERT

- Probabilistic estimate.
- 3 estimate are used :

  a → optimistic time.

  b → pessimistic time.

  m → most likely time.

Optimistic Time: If the execution goes extremely good.

Pessimistic Time: If the execution goes very badly.

Most Likely Time: Time estimate if execution is normal.

- The Probabilistic data for project activities generally follow beta distribution. The formula for mean (μ) and variance (σ^2) of beta distribution are given as:

  Mean $\mu = (a + 4m + b) / 6$

  And variance $\sigma^2 = [(b - a) / 6]^2$

The range for time estimate is from a to b.

- The expected project completion time is $\sum_i \mu_i$

  Where, μ_i → expected duration of critical activity
- The variance of the project completion time is $\sum_i \sigma_i^2$

  Where, σ_i^2 → variance of the i^{th} critical activity in the critical path.
- For the statistical analysis we need to know the probability of completing the project on or before due date (C).
- For this beta distribution is approximated to standard normal distribution whose statistics is given as:

  $Z = (x - \mu) / \sigma$

  Where, x → actual project completion time.

  μ → expected project completion time.

  σ → standard deviation of the expected project completion time.

  Therefore, P(x <= C) represents the probability that the project is completed on or before the time C.
- This can be converted into the standard normal statistics Z as:

  $P \{ [(x - \mu) / \sigma] <= [(C - \mu) / \sigma] \}$

  $= P \{ Z <= [(C - \mu) / \sigma] \}$

– EXAMPLE:

ACTIVITY	PREDECESSOR	DURATION (weeks)		
		a	m	b
A	-	6	7	8
B	-	1	2	9
C	-	1	4	7
D	A	1	2	3
E	A, B	1	2	9
F	C	1	5	9
G	C	2	2	8
H	E, F	4	4	4
I	E, F	4	4	10
J	D, H	2	5	14
K	I, G	2	2	8

a. Construct the project network.

b. Find the expected duration and variance of each activity.

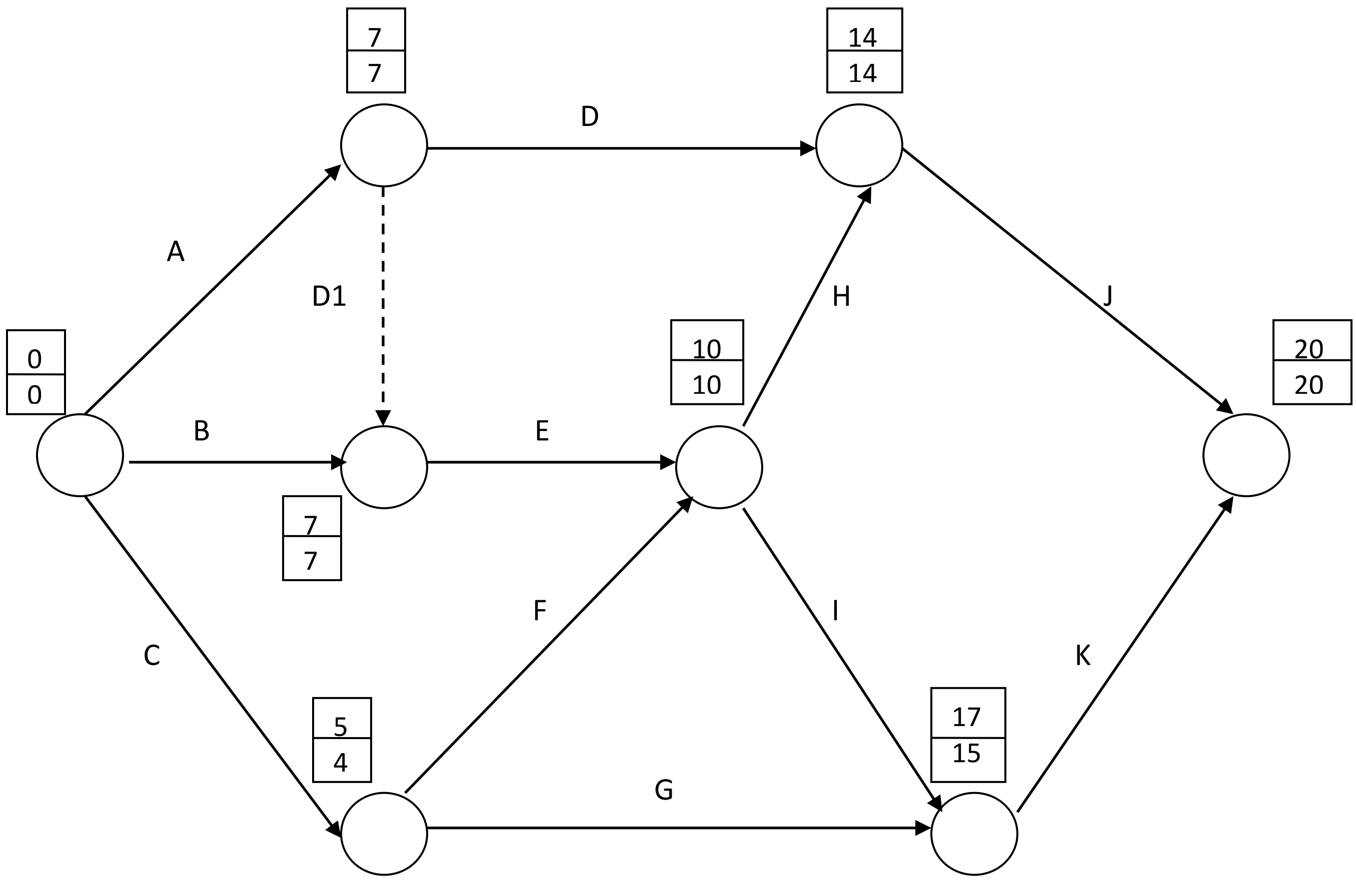
c. Find the critical path and the expected project completion time.

d. What is the probability of completing the project on or before 25 weeks?

e. If the probability of completing the project is 0.84, find the expected project completion time.

SOLUTION:

Step a.



Step b.

ACTIVITY	DURATION (weeks)			MEAN DURATION	VARIANCE
	a	m	b		
A	6	7	8	7	0.11
B	1	2	9	3	1.78
C	1	4	7	4	1.00
D	1	2	3	2	0.11
E	1	2	9	3	1.78
F	1	5	9	5	1.78
G	2	2	8	3	1.00
H	4	4	4	4	0.00
I	4	4	10	5	1.00
J	2	5	14	6	4.00
K	2	2	8	3	1.00

Step c.

Critical Path: A →D1 →E → H →J

Project completion time = 20 weeks.

Step d.

Sum of variance of all the activities on the critical path is:

0.11 + 1.78 + 0.00 + 4.00 = 5.89 weeks

Therefore, $\sigma = \sqrt{5.89} = 2.43$ weeks.

$$P (x <= 25) = P \{ [(x - \mu) / \sigma] <= [(25 - 20) / 2.43] \}$$
$$= P (Z <= 2.06) = 0.9803$$

Therefore, the probability if completing the project on or before 25 weeks is 0.9803.

Step e.

$$P(x <= C) = 0.84$$

$$P\{[(x - \mu) / \sigma] <= [(C - \mu) / \sigma]\} = 0.84$$

$$P(Ƶ <= [(C - 20) / 2.43]) = 0.84$$

From the standard normal table the value of Ƶ = 0.99, when cumulative probability is 0.84.

$$(C - 20) / 2.43 = 0.99$$

$$C = 2.44 \text{ weeks}$$

The project will be completed in 22.4 weeks (≈ 23 weeks) if the probability of completing the project is 0.84.