Inventory Control

Types:

- Raw material inventory.
- Work – in – progress inventory.
- Finished goods inventory.

Purpose:

The main functions of an inventory are:

- Smoothing out irregularities in supply.
- Minimizing the production cost.
- Allowing organizations to cope up with perishable materials.

Inventory Decision:

- When to order the inventory of an item?
- How much of an item to order when the inventory of that item is to be replenished.

Cost of Inventory Systems:

- Purchase price / unit.
- Ordering cost / order.
- Carrying cost / unit.
- Shortage cost / unit.

Frequent Order:

Cost of ordering is more, carrying cost is less.

Less frequent order:

Cost of ordering is less, carrying cost is more.

Optimal order size is the order size at which total cost is minimum.

Determination of the stock level of perishable items under probabilistic condition (stochastic order level system).

- In some cases, the demand on an item may not be constant and it will follow some probability distribution.

- 3 types of probability distribution
    1. Discrete Distribution.
    2. Uniform Distribution.
    3. Normal Distribution.
- Example: demand of a newspaper. For each unsold newspaper, there will be a penalty which is given by:

  Marginal cost of surplus / unit,

  S1 = Purchase Price per unit – Salvage value per unit.

  Similarly, for each shortage unit, there will be a penalty which is given by:

  Marginal cost of shortage / unit,

  S2 = Selling Price per unit – Purchase Price per unit.
- Let the generalized probability distribution of the demand of the item be a discrete distribution as given in a table:

Observation (i)	1 2 3 4 . . . . i . . . . n
Demand (D_i)	D_1 D_2 D_3 D_4 . . . . D_i . . . . D_n
Probability (p_i)	P_1 P_2 P_3 P_4 . . . . P_i . . . . P_n

Here, the optimal order size D_i* is determined by using the following relation:

$$P_{i-1} < S2 / (S1 + S2) < P_i$$

Where P_i is the cumulative probability of having demand up to D_i*.

EXAMPLE 1: The daily demand of the bread at a bakery follows a discrete distribution as given in table:

S.No.	1	2	3	4	5	6	7	8	9	10	11
Demand (D_i)	25	26	27	28	29	30	31	32	33	34	35
Probability (p_i)	0.2	0.11	0.10	0.09	0.08	0.12	0.14	0.05	0.04	0.04	0.03

The purchase price of bread is Rs 8 per packet.

The selling price = Rs 11 per packet.

If the bread packets are not sold within a day of purchase, they are sold at Rs 4 per packet to hotels for secondary use. Find the optimal order size of the bread.

SOLUTION:

Given purchase price / packet = Rs 8/=

Selling price / packet = Rs 11/=

Salvage value / packet = Ra 4/=

Therefore Marginal cost of surplus, S1 = 8 – 4 = 4

Marginal cost of shortage, S2 = 11 – 8 = 3

Therefore cumulative probability,

P = S2 / (S1 + S2) = 3 / (4 + 3) = 0.43

S.No.	1	2	3	4	5	6	7	8	9	10	11
Demand (D_i)	25	26	27	28	29	30	31	32	33	34	35
Probability (p_i)	0.2	0.11	0.10	0.09	0.08	0.12	0.14	0.05	0.04	0.04	0.03
cumulative probability (P_i)	0.2	0.31	0.41	0.50	0.58	0.70	0.84	0.89	0.93	0.97	1.00

From the table,

P3 < S2 / (S1 + S2) <P4 Or 0.41 < 0.43 < 0.50

Therefore the optimal order size is D4.

D4 = 28 bread packets.

EXAMPLE 2: In a railway private canteen, the daily demand for packed meals follows uniform distribution as given below:

P (x) = 1 / (250 + 150), 150 <= x <= 250

The cost of production per packed meals is Rs 9. The selling price is Rs 15 per packed meals. The surplus packets on each day are sold at RS 7 in a nearby public place. Find the optimal number of packets of meals to be prepared on each day.

<u>SOLUTION</u>:

Cost of production / packed meal = 9.

Selling price / packed meal = 15.

Salvage value / packed meal = 7.

S1 = 9 – 7 = 2

S2 = 15 – 9 = 6

Therefore cumulative probability,

P = S2 / (S1 + S2) = 6 / (2 + 6) = 0.75

Let Q* be the optimal production size. The cumulative probability distribution function of the uniform distribution when x = Q* is

$$\int_{150}^{Q*} \frac{1}{(250-150)}\,dx$$

$$= \frac{1}{100}\int_{150}^{Q*} dx = \frac{1}{100}\,[x]^{Q*}$$

$$= \frac{1}{100}\,[Q* - 150]$$

Therefore

$$\frac{1}{100}\,[Q* - 150] = \frac{S1}{(S1 + S2)} = 0.75$$

$$[Q* - 150] = 75$$

Therefore Q* = 225.

The optimal production size is 225 packed meals.

EXAMPLE 3: Alpha fish stall is planning for its optimal purchase quantity of a costly variety of fish. The daily demand of the fish follows normal distribution with a mean of 500 kg and standard deviation of 50 kg. The purchase price of the fish is Rs 120 / kg. The selling price is Rs 180 / kg. If the fish is not sold on the day of the purchase, it is sold to a dry fish manufacturing firm at Rs 100 / kg. Find the optimal daily purchase quantity of the fish.

SOLUTION:

Given,

Purchase price of a fish / kg = Rs 120

Selling price of a fish / kg = Rs 180

Salvage price of a fish / kg = Rs 100

Therefore Marginal cost of surplus, S1 = 120 – 100 = 20

Marginal cost of shortage, S2 =180 – 120 = 60

Cumulative probability,

P = S2 / (S1 + S2) = 60 / (20 + 60) = 0.75

Mean μ = 500

Standard deviation σ = 50

Let Q^* → optimal delay purchase quantity of fish.

Therefore, the standard normal statistics, Ƶ for the demand is,

Ƶ = $Q^* - \mu / \sigma$

= Q^* - 500 / 50

For a Cumulative probability of 0.75, Ƶ = 0.675

Q^* = 533.75 kg ≈ 534

Therefore, the optimal daily order size of the fish is ≈ 534 kg.

Replacement Theory

- What should be the optimal replacement period?
    a. Replacement policy for equipment which deteriorates gradually.
    b. Replacement of items that fail suddenly.
- The following costs are associated with an equipment over a given time period, n

  C → purchase price of equipment.

  S → scrap value of equipment at the end of n years.

  M_n → maintenance cost of the equipment in n years.

  Therefore, the total cost for owning and maintaining the equipment for n years:

  $$T(n) = C - S + \sum_{i=1}^{n} M_n$$

  And average cost,

  $$A(n) = 1/n\,[C - S + \sum_{i=1}^{n} M_n]$$

  In this case, the optimal replacement period would be the one in which A (n) is minimum.
- EXAMPLE: A firm is using a machine whose purchase price is Rs 13,000. The installation charges are Rs 3,600 and the maintenance cost in various years is given as:

YEARS	1	2	3	4	5	6	7	8	9
COST (Rs)	250	750	1000	1500	2100	2900	4000	4600	6000

The firm wants to determine after how many years the machine should be replaced on economic considerations assuming that the machine replacement can be done only at the year ends.

SOLUTION:

Cost of machine C = purchase price + installation charges

= 13000 + 3600

= 16,600

S = 1600

Determination of optimal replacement period

YEARS **(n)**	Maintenance cost **(M_i)**	Cumulative Maintenance cost **($\sum M_i$)**	**C - S**	**T (n)** = $\sum M_i + (C - S)$	**A(n)** = T (n) / n
1	250	250	15000	15250	15250
2	750	1000	15000	16000	8000
3	1000	2000	15000	17000	5667
4	1500	3500	15000	18500	4625
5	2100	5600	15000	20600	4120
6	2900	8500	15000	23500	3917
7	4000	12500	15000	27500	3929
8	4800	17300	15000	32300	4038
9	6000	23300	15000	38300	4256

The machine may be replaced after 6 years.

REPLACEMENT THEORY FOR ITEMS WHICH COMPLETELY FAIL.

PROBLEM: The failure rates of 1,000 street bulbs are given as:

END OF MONTH	1	2	3	4	5	6
PROBABILITY OF FAILURE	0.05	0.20	0.40	0.65	0.85	1.00

The cost of replacing an individual bulb is Rs 60. If all the bulbs are replaced simultaneously it would cost Rs 25 per bulb. Any one of the two options can be followed to replace the bulbs.

1. Replace the bulbs individually when they fail (individual replacement policy).
2. Replace all the bulbs simultaneously at fixed intervals and replace the individual bulbs as and when they fail during the fixed interval (group replacement policy).

Find the optimal policy, if group replacement is optimal then find at what equal intervals all the bulbs are replaced?

SOLUTION:

Number of bulbs in the colony, N_o =1,000

Let p_i → probability that a bulb fails during the i^{th} month of its life.

Hence, $p_1 = 0.05$

$P_2 = 0.20 – 0.05 = 0.15$

$P_3 = 0.40 – 0.20 = 0.20$

$P_4 = 0.65 – 0.40 = 0.25$

$p_5 = 0.85 – 0.65 = 0.20$

$p_6 = 1.00 – 0.85 = 0.15$

Let N_i be the number of bulbs replaced at the end of the i^{th} month.

If k is the maximum number of periods of life that a unit can have, the number of failures, N_i in any given period i, can be expressed as:

$$N_i = N_o p_i + \sum_{t=1}^{i-1} N_{i-t}\, p_t \qquad \text{for } i \leq k$$

$$N_i = \sum_{t=1}^{k} N_{i-t}\, p_t \qquad \text{for } i > k$$

$N_o = 1000$

$N_1 = N_o\, p_1 = 50$

$N_2 = N_o\, p_2 + N_1\, p_1 = 153$

$N_3 = N_o\, p_3 + N_2\, p_1 + N_1\, p_2 = 215$

$N_4 = N_o\, p_4 + N_3\, p_1 + N_2\, p_2 + N_1\, p_3 = 294$

$N_5 = N_o\, p_5 + N_4\, p_1 + N_3\, p_2 + N_2\, p_3 + N_1\, p_4 = 290$

$N_6 = N_o\, p_6 + N_5\, p_1 + N_4\, p_2 + N_3\, p_3 + N_2\, p_4 + N_1\, p_5 = 300$

<u>Calculation of individual replacement cost.</u>

Expected life for each bulb = $\sum_{i=1}^{6} i\ p_i$

$= 1 * 0.05 + 2 * 0.15 + 3 * 0.20 + 4 * 0.25 + 5 * 0.20 + 6 * 0.15$

= 3.85 months

And average no of failure per month = N_o / 3.85

= 1000 / 3.85 ≈ 260

Therefore, the cost of the replacement = no of failure per month * individual replacement cost per bulb

= 260 * 60 = Rs 15600/=

<u>Calculation of group replacement cost.</u>

Cost per bulb when replaced simultaneously = Rs 25.

Cost per bulb when replaced individually = Rs 60.

(a) END OF THE MONTH (n)	**(b)** COST OF REPLACING 1000 BULBS AT A TIME	**(c)** COST OF REPLACING BULBS INDIVIDUALLY DURING GIVEN REPLACEMENT PERIOD	**(d)** TOTAL COST d = b + c	**(e)** AVERAGE COST PER MONTH e = d / a
1	25000	50 * 60 = 3000	28000	28000
2	25000	(50 + 153) * 60 = 12180	37180	18590
3	25000	(50 + 153 + 215) * 60 = 25080	50080	16693
4	25000	(50 + 153+ 215 + 294) * 60 = 42720	67720	16930
5	25000	(50 + 153+ 215 + 294 + 290) * 60 = 60120	85120	17024
6	25000	(50 + 153+ 215 + 294 + 290 + 300) * 60 = 78120	103120	17187

Economic life of bulb = 3 months

The group replacement period = 3 months

Individual replacement cost / month = Rs 15,600.

Group replacement cost / month = Rs 16,693.

In this case *Individual replacement policy is the best*, and hence all the bulbs are to be replaced as and when they fail.