NOTE: The following lecture is a transcript of the video lecture by Prof. Debjani Chakraborty, Department of Mathematics , Indian Institute of Technology Kharagpur.

Golden section Method

Now, golden section method. Today I am discussing that method and that method is applicable for finding out optimal solution, for 1 dimensional non-linear programming problem. Now, golden section method is a method like other elimination techniques like Fibonacci method, Dichotomic search and other searching techniques, were we are eliminating the given region, given interval of uncertainty iteratively. But here the golden section method, there are certain things to be mentioned .There are very special for this method is that, this method is totally is totally depend on one ratio that is called the golden ratio. And there is a history of this golden ratio.

I will come to that. Before to that, I just want to tell you that golden section method, it has certain advantages and certain disadvantages as well. The basic assumption for applying this method is that function must be unimodal, within the given interval of uncertainty. If the function is not unimodal, if the function is multimodal; that means, several maxima local maxima minimas are there within the interval of uncertainty, then we have to break the interval into smaller interval. So, that in the one part of the interval function is unimodal.

Then we will apply the golden section method, for obtaining the optimal solution whether local maxima or minima, and at the end we can approach for the global optimal solution. This is one part. The next part is that, golden section method is similar to the Fibonacci method, but one limitation, one restriction is that. Not limitation one relaxation rather is that, number of experiments are not predefined. We can, we are in the process, in the process itself we can just select the number of experiments, and we will just prepare in such a way that, we will do the large number of experiments to get the better approximation. And number of experiment, we can just stop according to our tolerance limit and show you in the next, how to adopt that fact in the process.

That is why, with that let us start the golden section method. Since this is the iterative process again, our task is to learn the whole process of the iteration and I will just show you how stepwise the process is running, from the initial interval of uncertainty to the final uncertainty. But everything depends one thing that is called the golden ratio. And this is by convention, we are writing the golden ratio with gamma.

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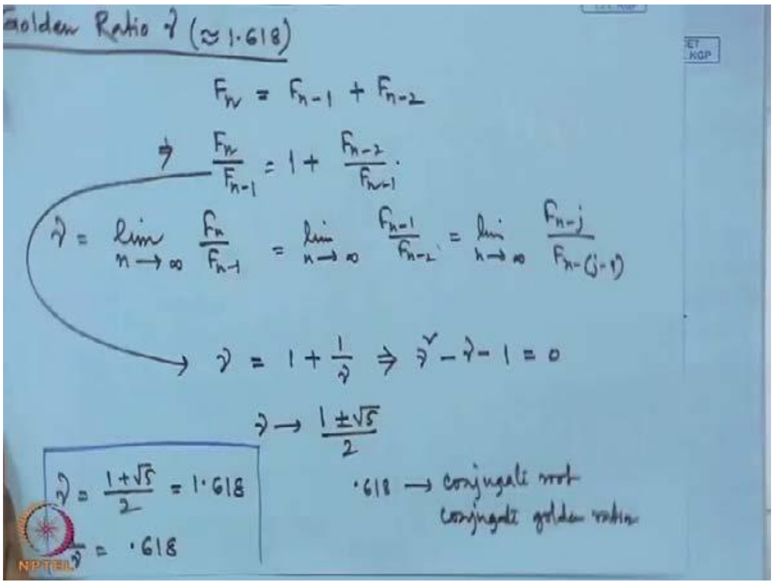

And this ratio, first let me tell you the value of this ratio. We will see this value is almost same as 1.618. I will first say, how this value is coming, after that we will use this ratio in our iteration process. The golden ratio, again little more its little dependent on Fibonacci method, Fibonacci numbers not method, Fibonacci numbers. As we know, in the Fibonacci numbers, F n is equal to F n minus 1 plus F n minus 2. That is, any Fibonacci number is sum of the previous 2 Fibonacci numbers in the sequence. Then, from here if we just divide both side with F n minus 1, then we are getting F n divided by F n minus 1 is equal to 1 plus F n minus 2 divided by F n minus 1. And after that, as I said that, we are ready to do large number of experiments and we will get ratio from this equation only, by taking limit n tending to infinity. And by considering gamma is equal to limit n tending to infinity F n by F n minus 1.

If this is gamma value, then we will see that, this will be same as F n minus 1 divided by F n minus 2. This will be same as limit n tends to infinity F n minus z divided by F n minus z minus 1. If this is so, then this equation can be written as gamma. This is by considering n tending to infinity, F n by F n minus 1 is equal to gamma, is equal to 1 plus and this is 1 by gamma. Because these value is gamma only. Then this is 1 by gamma and from here we are getting one second degree equation. Gamma square minus gamma minus 1 equal to 0. And since, this is polynomial of degree 2, we will apply the process to get the root of this equations, by considering minus p plus minus root over b square minus 4 a c divided by 2 a is we just consider, we will get 2 roots of gamma.

1 plus minus root 5 divided by 2, plus minus root 5 divided by 2. And if we just consider the positive value of this gamma, the positive value; that means, 1 plus root 5 by 2, this value will be almost similar to 1.618. And just look at the thing that, if we consider gamma is equal to limit n tending to infinity F n by F n minus 1, we are getting the value of the gamma as 1.618. And this is the golden ratio value and since we are getting another root here, that is the negative root and that negative root would be is equal to minus 0.618. And if we just considered the root and we will say that the other root 0.618, this is the conjugate root. Conjugate root, rather conjugate golden ratio.

And what is the value for this. This is equal to 1 by gamma only. That is why we are getting gamma is equal to this and 1 by gamma is equal to 0.618. These 2 facts, we will use in the next in our process, in our iteration process. Now gamma has some physical, has some historical background, this golden ratio. If we just look back the history of the golden ratio we will see that.

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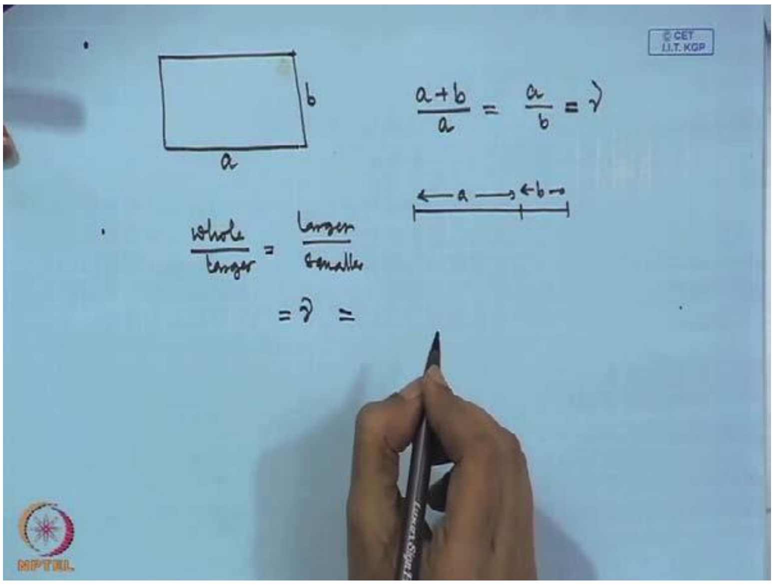

If we consider, actually ancient Greek architects, they believed that if we consider a room, where the size of the room is like this. The larger side of the room is b a and the smaller side of the room is b and it has been it was, it was a believe by the Greek architect that the room is having the pleasant property, if the ratio of the whole divided by larger, ratio of the whole tool larger is equal to its ratio of the larger to smaller. If we consider, that ratio we will be always equal to gamma. That is why if we consider, if we were believing that if we consider the room, the construction in such a way that the larger side plus the smaller side divided by the larger side is equal to gamma, or the larger side divided by smaller size is gamma, there room will be nice room.

But this is one of the fact, and another fact, if we go back to the history of the golden ratio, it has been mentioned in the Euclid geometry that, if it is divide line segment into 2 unequal parts. Then we will see that length of the whole; that means, we are considering the whole line, the larger part is a and the smaller part is b. Then the length of the whole divided by the larger part is equal to length of the larger divided by smaller. That is the ratio.

Again it would be is equal to gamma. And this golden ratio has some significant and other cases also we will see in the natural objects. People are using golden ratio for selection of the size of the postcard, size of the white screen t v and they are considering the sides of object in such a way that the total, that ratio of the whole. That is the smaller plus larger is equal to larger by smaller, then it will have a nice pleasant nice look of that object.

This is the beauty of golden ratio. Now, let us not go into detail about that. I will just, we will use this ratio in our iteration process in the optimization technique in the next. The only fact we are considering the golden ratio is gamma and we are considering gamma is coming from the Fibonacci number. By considering the number of experiments in finite and the value is 1.618.

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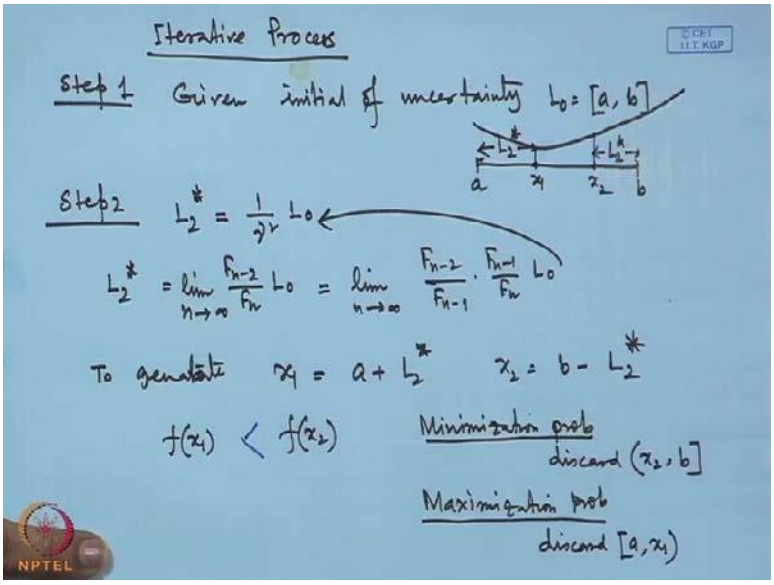

Let us start our iteration process. And objective of the process is to get the minimum or the maximum of non-linear function. That is single dimensional, considering 1 variable. Function can be discontinuous as well, for that case also it is applicable. The step 1, it is only given the initial interval of uncertainty. Number of experiments is not needed for us, that is why we will start with our process from initial level of uncertainty L naught. That is from, let us consider it from a to b. Step 2, we will consider 2 experiments: x 1 and x 2, within a to b in such a way that, it is L 2 star distance upper from both the ends. What is the value of L 2 star ? Value of L 2 star is equal to 1 by gamma square L naught.

How we are getting this value? If I just go back to the Fibonacci method, we saw that the L 2 star is equal to F n minus 2 divided by F n L 0. Now, here since this is the golden section method, we are considering limit n tending to infinity, large number of experiments we want to do. That is why let us do certain simplification here. We are considering F n minus 2 divided by F n minus 1 into F n minus 1 by F n and L 0. As we know, limit n tending to infinity F n by F n minus 1 is 1 by gamma. That is why, this is 1 by gamma, this is 1 by gamma. That is why, all together its coming as 1 by gamma square L naught.

Once we are getting so, then our task is easy to generate x 1 and x 2. x 1 is equal to a plus L 2 star and x 2 is equal to b minus L 2 star. And we will find out the functional values, f x 1 ad f x 2. And if we see that, f x 1 is lesser than f x 2 for minimization problem we will conclude that, say the function is this one, this is my function .What we see here, at x 1 whatever functional value we are getting x 2 the functional value is higher. That is why, we are having this relation here and if this is the minimization problem. For the minimization problem, what we will do? We will discard the interval x 2 to b. And what about for the maximization problem?

Since, the function is unimodal, the maximum value cannot lie here. Because it is having only 1 maxima in this function. That is why the function could be like this, in this way. Then in that case, we will discard other part. That is, from a to x 1. Once we are reducing the interval, my next interval of uncertainty would be either from a to x 2, or from x 1 to b. All right, that is why I will move to the next step.

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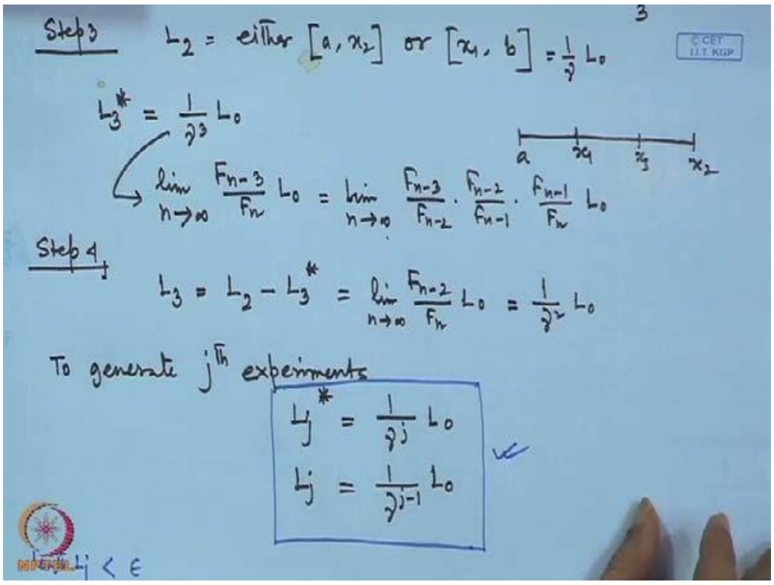

Step 3, we will have the new interval of uncertainty. It would be either from a to x 2 or from x 1 to b. Now what is my next task? My next objective is to find out a to x 2 in between x 1 is there. .Again we will we will find out another approximation, we will do another experiment, which will lie within the point a to x 2 in the first case. Say x 3 is here, how to generate x 3 ? For that thing, we have to find out L 3 star. L 3 star would be again would be is equal to 1 by gamma cube L naught. How we got it? Again with the similar process, that can be written as limit n tending to infinity just like the Fibonacci method F n minus 3 divided by F n L 0. That can be written as limit to infinity F n minus 3 divided by F n minus 2, F n minus 2 divided by F n minus 1, F n minus 1 divided by F n L 0.

As we know limit n tending to infinity F n by F n minus 1 gamma, then it must be 1 by gamma. This is 1 by gamma, this is 1 by gamma. That is why we are going to gamma, 1 by gamma q L naught. Now just see the process, we have considering 1 by gamma q. What is the value of gamma? Here we are considering value of gamma as 1.618. Rather 1 by gamma is 0.168, as we got it from the golden ratio .Then what is the next interval of uncertainty, we will move to the step 4. We will get the next interval of uncertainty. That is L 3.What is the value of L 3? Value of L 3 would be, again that would be is equal to L 2 minus L 3 star. Rather this would be is equal to F n minus 2 divided by F n L 0, limit n

tending to infinity .If we can see there this way and that would be is equal to 1 by gamma square L 0.

In the previous case, L 2 was is equal to 1 by gamma L 0. L 3 is getting gamma square L 0. That is why, in general we can say that, to generate the j th experiment, we are doing large number of experiments. If we do large number of experiments; that means, we are approaching to the optimal solution in a better way, rather we are getting interval of uncertainty with smaller and smaller width and that is more desirable to us. That is why, to generate the j th experiment, we can generate the formula for the iteration as L j star is equal to, very nice to remember, 1 by gamma to the power j L naught. And the j th value of uncertainty would be is equal to 1 by gamma to the power j minus L naught. We need not use any Fibonacci number anything. Just to understand, how we are reaching to this formula we have just employed the definition of gamma here.

Otherwise, without using the definition very easily we can generate this formula for golden section method. And by just supplying the value for j is equal to 1, 2, 3, 4 we can move to the say, in the respective steps of the process. And where to end, that is the more desire, more important thing for us. That is why we conclude in this way. We will just see the length of the interval final interval of uncertainty after the j th experiment. If we see that, length of L j is lesser than very small positive value epsilon, then we will conclude that, stop the iteration process and declare that L j as the final interval of uncertainty. And certainly, the middle point of the final interval of uncertain would be the optimal solution of the problem. That is the whole process.

As you have seen the process is a very easy to handle, computation process is very easy. Only thing is that, we have to use the golden ratio value and we have to remember this formula. Let us apply this technique for certain problem in the next and let us see how the iteration can proceed further.

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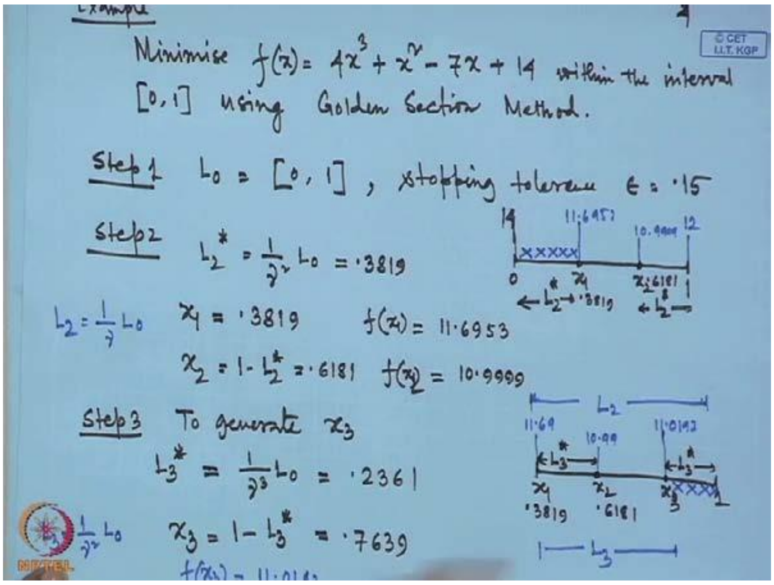


Let us consider the example, minimize f x is equal to 4 x cube plus x square minus 7 x plus 14 within the interval 0 1, using golden section method. Now, if we just see the property of the function, function is given to us. Initial interval of uncertainty is given. It is assumed that the function is unimodal in the given interval otherwise, if we just draw the graph of this function we will see the function is a very much unimodal. It is having only one minimum value within 0 to 1. That is why, let us start our process. What is the first step?

First step is that L naught is given to us, that is the initial interval of uncertainty. The length of L naught is 1 and another thing we may consider, otherwise we can may not consider also. As a stopping criteria of the iteration process, we are considering a stopping tolerance. Epsilon is equal to 0.15. We may consider, we may not consider. Once we are considering the stopping tolerance; that means, in the process we will just check what is the length of the L j and if it is lesser than this, then we will conclude the process.

We may not consider, also in the process we can just generate our stop stopping criteria. Let us move to the step 2. What is step 2? We are having 2 points. What we see at the functional value at 0 and 1. At 0, the functional value is coming 14 and at 1 the functional value is 12. We need to generate 2 experiments in between 0 to 1. 1 is x 1 another one is x 2. How to get x 1 and x 2? It must be L 2 star distance apart from both the ends. That is by the consideration of the golden section method. What is L 2 star? We have just considered L 2 star is equal to 1 by gamma square L naught. We know 1 by gamma is equal to 0.168. That is why, it must be is equal to 0.618 to 0.618 into L 0. That is equal to 1 and we will get the value as 0.3819.

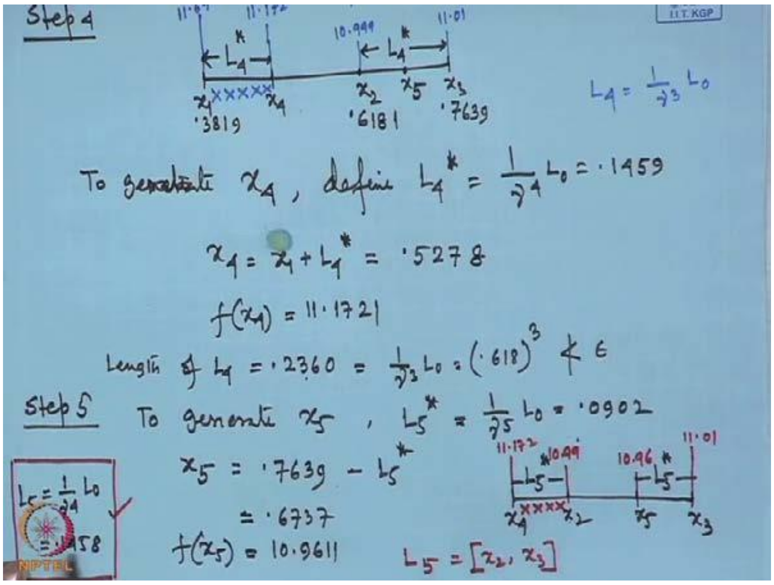
That is why once we are getting this L 2 star, x 1 would be is equal to 0.3819 and x 2 would is equal to 1 minus L 2 star. It is equal to 0.6181. This is equal to 0.3819 and x 2

is 0.6181. Let us see the functional value at x 1 and x 2 .Functional value at x 1, its equal to 11.6953 and if we just substitute the value x 2 in the given function, we will see the functional value we will come as 10.9999. And if we draw it here, what we see at x 1 it is having higher value 11.6953 and x 2 its lower value 10.99.

Since function is unimodal, we are we want to get the optimal minimum value. That is why we will discard this interval. Because f x 1 is higher than f x 2. We will discard this interval and my next interval of uncertainty would be x 1 to 1. In between x 2 will be there. x 1 is equal to 0.3819 and x 2 is 0.6181 and this is my the last b value. Now my next step, we are moving to the next step. We have to generate the next approximation for the optimal solution, that x 3. How to generate x 3? For that thing we need to generate L 3 star. And we will get L 3 star is equal to, again the same gamma cube L naught and x 3 will be such that it would be the current 2 experiments. That is, x 2 and x 3 both will be L 3 star distance afford from both the ends.

That is why, this would be L 3 star, this length again e this length will be L 3 star. All right. That is why we will get L 3 star is equal to 0.2361. Certainly, we will get x 3 is equal to 1 minus L 3 star and this is coming as 0.7639. And let us see the functional values again and one thing I must mention here is that, if we just see the interval of uncertainty after second step, the whole length is L 2, this length. And this L 2 would be is equal to according to my formula 1 by gamma L naught. Here we will we are moving to the next step. We will move to L 3. We will see L 3 would be equal to 1 by gamma square L naught. We want to achieve this value, let us see.

At point x 1 functional value is 11.6953. At point x 2, it is 10.99 and if we just go for f x 3 this value is coming as 11.0193. This is the value for this 11.0193. Then, since the function we want to achieve the minimum of this function. That is why, certainly we will discard this region in the next iteration. That is why my L 3 will be from this point to this point. That is, from x 1 to x 2. In the next step, let us move to the next step. And if we just see the length of L 3 it will come as 1 by gamma square L naught. (Refer Slide Time: 28:15)



Step 5, step 4, my interval of uncertainty is from x 1 to x 3. The value of x 1 is 0.3819, x

3 is equal to 0.7639. In between we are having x 2, this is 0.6181. We have to generate the next experiment x 4. For that, to generate x 4 we need to define L 4 star. L 4 star would be is equal to 1 by gamma 2 the power 4 L naught. That would be 1.1459.

Then x 4 will be is equal to and again the value of x 4 will be such that current 2 experiments that is x 4 and x 2, that would be L 4 star distance apart from both the ends. If we just see the distance between x 2 and x 3, we will see that this is L 4 star. Then, certainly x 4 must be here. It would be L 4 star distance apart. That is why, x 4 would be equal to x 1 plus L 4 star. That is if we just add together we are getting x 4 is equal to 0.5278. And with the functional value x 4 is equal to 11.172.1. Then let us draw the values in the graph. We are having 11.69 something. At x 4 it is 11.172. At x 2, it is 10.9999 and it is 11.01.

We are considering the assumption that is the function is unimodal, function is having 1minimum within the interval. That is why, certainly we will discard this interval in the next step. And we are we will move to the next interval of uncertainty L 4. Only this part and we will see that fact that length of L 4 would be is equal to 1 by gamma cube L 0. That we can check very easily. L 4 is from x 4 to x 2, x 3. We can just see the length. Now, we will generate .Since L 4 the length of the L 4 is equal to 0.2360 which is equal to gamma cube L 0, rather 0.618 to the power cube. This value is not less than epsilon. That is why, we have to move to the next iteration again.

Because this length is not small enough. To consider as a final interval of uncertainty we are moving to the next step 5. In the step 5, we will generate the x 5. To generate x 5, we will calculate L 5 star and we will see L 5 star would be is equal to 1 by gamma to the power 5 L 0, and this value is 0.0902. Once we are considering so, we will see x 5 will be is equal to 0.7639. That is again the same logic. We will consider x 5 in such a way that the current two experiments, if we just if we just see the L 4 interval, the one end is x 4 another end is x 3. One approximation is there, that is x 2 and another current approximation we want to make that x 5.

Then, these 2 experiments must be L 5 star distance apart from both the ends. That is why, x 5 if we just see the if we just see the distance between these two x 2 and x 3. We will see that it will be is equal to 0.0902 and x 5 will be is equal to. So, that is why this will be minus. Certainly one thing is that, this is this is x 5 will be here. And this distances, we will see if I just draw the graph once more here with the point x 4, x 2, x 3. Then we will see this is L 5 star, this distance. And we will consider L 5 in such a way that it is in between. And this value will come as 0.6737 and with the functional value x 5 is equal to 10.9611.

And if I just draw the function once more here. And another thing, if I just see the length of this interval L 5, we will see that it would be is equal to 1 by gamma to the power 4 L 0 and that is coming the value is 0.1458. That is why, this is lesser than epsilon we have considered epsilon, we will consider as one point 0.15, that is why we can conclude 1 epsilon. That is, we are stopping up our iteration process. But let us see, which part of the interval will discard in this range. We are having x 4 as the 11.172, x 2 as 10.99, x 5 as 10.96, lesser than that and x 3 as 11.01.

What we see, we see that this is the less x 5 f x 5 is less than f x 2. That is why, we will discard this interval, since the problem is the minimization problem. And we will declare the final interval of uncertainty L 5 is equal to from x 2 to x 3 and this length will be is equal to 0.1458. And this is the whole process of golden section method. Now, whatever we have done till date, these are the elimination techniques, we have considered starting from the exhaustive search, dichotomous search interval halving Fibonacci method, golden section method.

Now, since we have learn all the methods one by one we need to do the next analysis. Which method is the best method among these? Which elimination technique gives us the better result. Rather with lesser number of iteration, we will get the result with lesser number of calculation lesser number of experiments we are getting the optimal solution. That study we have to do for that thing. We have to measure the, we have to measure the efficiency of the different methods. That is why, we are moving to the next that is all about golden section method. We are moving to the next that is efficiency of the region elimination technique.

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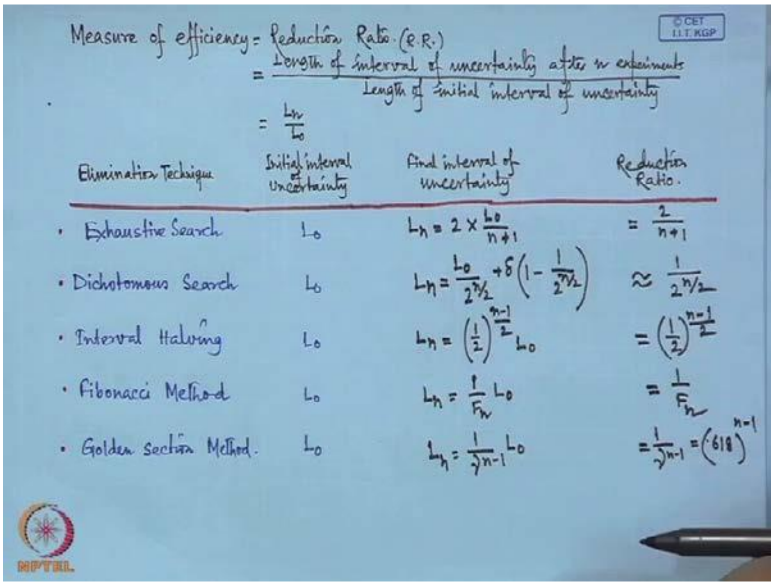

If L 0 is the initial interval of uncertainty and if we just do n number of iterations, rather n number of experiments if we just generate 1 by 1, as we have I have shown you that then L n will be the final interview of uncertainty. And measure of efficiency will be calculated as length of the interval of the uncertainty after nth experiment divided by length of the initial interval of uncertainty, rather this will be is equal to L n by L 0. This is also named as reduction ratio and we call it as r dot r.

Let us see, what are the different values for different searching techniques one by one. If I just see this table, we have considered here different elimination techniques starting from exhaustive search, dichotomous search, interval halving, Fibonacci method, golden section method. And we are having the initial interval of uncertainty L naught. We will see the final interval of uncertainty. Then we will go to the reduction ratio. One by one, we will just move. In the exhaustive search method, we have seen that L n was coming is equal to 2 into L 0 by n plus 1. That is why, the reduction ratio is coming as L n by L 0.

Since this is so, then this is equal to 2 by n plus 1. What is n? n is the number of experiments. If we do 2 number of experiments that is very less for iteration. That should be 2 by 3, if we do 6 number of experiments that would be 2 by 7. That is the reduction ratio. For the method which is having lesser reduction ratio, that method is much efficient for us. Let us go for the dichotomous search. If we just see the dichotomous search technique, we will see that L n can be calculated as in the dichotomous searching there is another not only L naught there is another parameter also involved. That is delta, generally we are considering the value of delta as very small.

But still, if we just find out the final interval of uncertain this would be equal to L n by 2 to the power 2 plus delta into 1 minus 2 to the power 1 1 by 2 to the power n by 2. This is the final interval of uncertainty. If we just see the reduction ratio L n by L 0, delta is very small, let us say we can ignore this part. That is why, we can see it is almost same as 1 by 2 to the power n by 2. That is for the dichotomous interval. What about halving interval halving? Interval technique L n is equal to, we have learned this is half to the power n minus 1 by 2 L 0. And if I just see the reduction ratio, this is equal to 1 by 2 to the power 1 n minus 1 by 2.

Fibonacci method, we have learnt L n is equal to 1 by F n L 0, rather this is 1 is 1 is nothing, but f 1 L 0. That is why, L n by L 0 would be is equal to 1 by F n. What about golden section method? Just now, we have received that the golden section method L n can be calculated as 1 by gamma to the power n minus 1 L 0. That is why, we can say this is equal to 1 by gamma to the power n minus 1, rather this is equal to 0.618 to the power n minus 1. This is, we could see this is one at a glance we could see the different reduction ratios of the different searching technique. Let us move to the specific value of this reduction ratios for different cases.

For 2 cases, let us consider the thing. One is that we are considering the reduction ratio when we want to achieve 10 percent of exact value. That is, we are we are fixing the tolerance of the error would be 10 percent on the either side. We are saying either side because as we as I have said that, the final interval of uncertainty is the interval where we say the optimal may lie. But if I want to have the optimal value within that interval, always that would be the middle value of that range.

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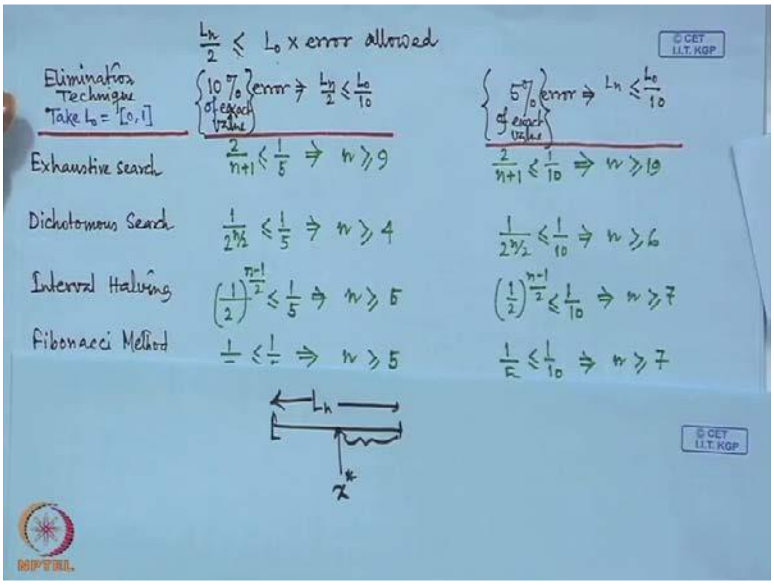
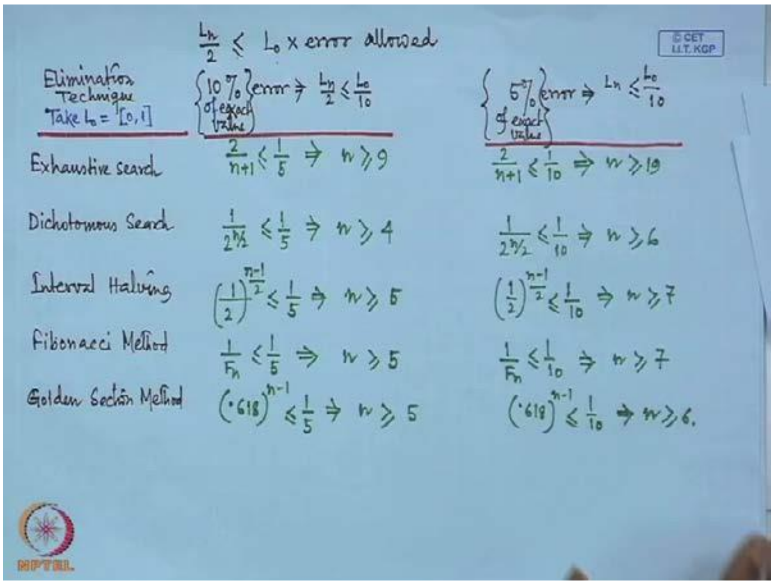


That is why if L n is the interval of uncertainty. This is the L n for us. Then optimal must be at middle point. x star is the middle point. That is why, if I want to have the 10 percent of the 10 percent of the exact value, rather 10 percent of the initial interval of uncertainty if I want to have it in other way, then L n by 2 must be less than is equal to L naught into error we are allowing. This is the calculation we are just doing. We are considering again all the techniques one by one. We are considering the elimination technique by considering L naught is equal to 0 to 1 for specific values we are just working. And 10 percent of the exact value we are considering as an error. That is why, L n by 2 must be less than equal to L naught by 10. And if we consider 10 percent of the x z value as the error limit.

Then, it would be L n by 2 must be less than is equal to L naught into 5 by 100. That is why, we are getting L n less than is equal to L naught by 10. If we just calculate for every for all the searching techniques one by one. For exhaustive search, as we know the reduction ratio L n by L 0 is equal to 2 by L plus 1. That is why, 2 by n plus 1 must be less than is equal to 1 by 5. If I just go 2 will come here and it will be 1 by 5 and from here we can calculate the value of n greater than equal to 9.

That means, we need to do at least 9 number of experiments to get this level of accuracy. And for the other thing if we consider 5 percent of exact value we have to have at least 19 number of experiments rather 19 number of approximation. 19 approximations we have to make to get the accuracy of 5 percent of exact value, this is for exhaustive search. This is for dichotomous search. As this is the reduction ratio, 1 by 2 to the power n by 2 must be less than equal to 5, we can calculate n must be greater than equal to 4. If I want at 5 percent exact value, then we have to do at least 6 number of experiments. (Refer Slide Time: 44:07)



Similarly, for others if I just see for the interval halving technique we are having n must be greater than equal to 5, but for 5 percent accuracy we must do atleast 7 number of experiments. For the Fibonacci method, this is the reduction ratio 1 by F n. Just now, I have showed you. Then, from here we can deduce that n must be greater than equal to 5, but if I want to have the 5 percent accuracy n must be greater than equal to 7. And the golden section method ,we are getting n must be greater than equal to 5 for 10 percent of exact value, if this is the error tolerance and if want 5 percent of exact value, then n must be greater than equal to 6. This is some calculation I made, if I want to do more exact calculation we will see that that Fibonacci method is the best method among all these searching methods.

Why it is best because computational process is very easy, this is one part. But the most important part is that we need to do less number of experiments for even for weighted accuracy. If I want to have 1 percent of the exact value, error limit if we just tolerate 1 percent in either side of the exact value, then we need to do less number of experiments than any other method. But one thing must be mentioned here also that golden section method, this is also very much similar to the Fibonacci method. That is why, golden section method is also very good method for efficient method, effective method for finding out the optimal solution for non-linear optimization problem, single dimensional optimization problem.

And one thing it is known that Fibonacci method and the golden section method, golden section method is almost identical to Fibonacci, because this is very much asymptotic to the Fibonacci method. Because as we know that, if I just increase the number of experiments n tending to infinity then, Fibonacci method is same as the golden section method. That is why, these are all the region elimination techniques I just wanted to discuss, all just at a glance this is the comparison of all methods.

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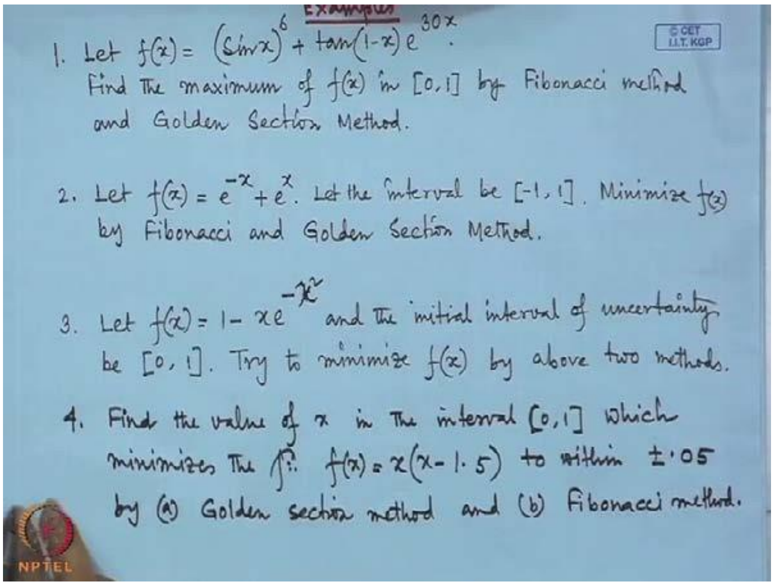

Now, let us move to the next that is the problems which you can address for practice. These are the examples we can consider. We can consider the function f x as sin x to the power 6 plus tan into tan 1 minus x e to the power 30 x, where function is in between 0 to 1 we need to find out the maximum of this function by Fibonacci method and golden section method both. You can do it. And we can consider another function minimization of f x which is equal to e to the power minus x plus e to the power x and the function is defined within the interval minus 1 to 1, function must be unimodal within this interval then only the Fibonacci method and golden section method both are applicable,that is why that can be trained.

We can see the optimal solution for this function. That is the minimum of this function using Fibonacci method and golden section method separately. Let us consider another function, that is function f x and 1 minus x into e to the power minus x square, we have to minimize function and both the functions can be minimized using Fibonacci method and golden section method. That is all about the region elimination technique.

Let us take another example as well. This example I am I have considered from the book written by professor S S Rao, engineering optimization engineering optimization theory and practice. And this is a problem for us .Find the value of x in the interval 0 1 which minimizes the function f x is equal to x into x minus 1.5 and 1error limit is given also here to within plus minus 0.05 that much error is allowed, by golden section method and by Fibonacci method as well.

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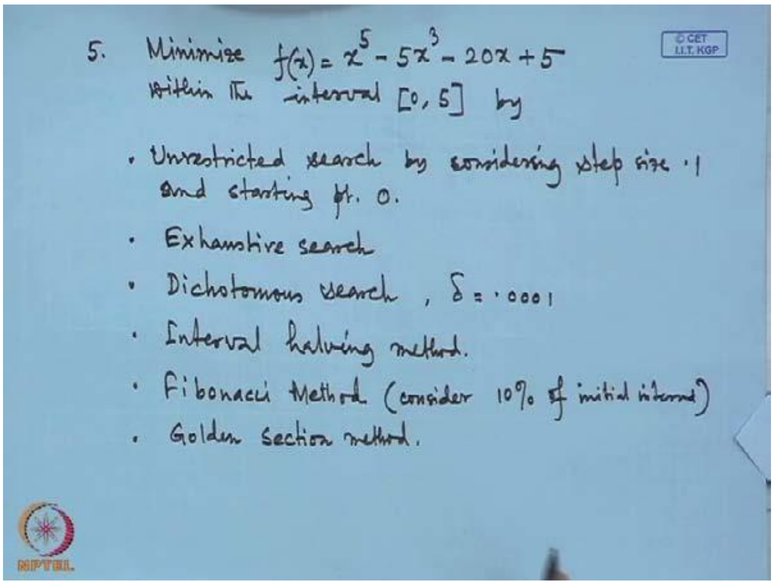

Let us consider another example in the next. That is the function I am just writing that can be all the methods can be applied here. Just see the, we can you can do the comparison of the processes. Minimize f x is equal to x to the power 5 minus 5 x cube minus 20 x plus 5 within the interval 0 to 5 and the methods you can adopt, unrestricted search method by considering the, by considering step size 0.1 and starting point 0.x equal to 0.

Then we can apply the exhaustive search within the whole range. We can apply the dichotomous search. And as we know, there is a parameter for dichotomous search we can consider a very small value for delta that 0.001. We can apply the interval halving technique, apply Fibonacci method, apply the golden section method. And Fibonacci method, here you can consider a 10 percent of initial interval of uncertainty. That is 5. That is an error limit and golden section method according with a small interval size. And that is all about the elimination technique. We can implement for solving non-linear one dimensional non-linear programming problem.

Thank you for today.