NOTE: The following lecture is a transcript of the video lecture by Prof. Debjani Chakraborty, Department of Mathematics , Indian Institute of Technology Kharagpur.

Interpolation Methods

Now, today's topic is Interpolation Method for solving non linear programming problem, now interpolation methods are very useful methods, and very efficient method the for solving non-linear optimization problem. And it is one of the very important line search technique for solving non-linear, now there are various interpolation methods are available. One of that is the quadratic interpolation method, and other one is the cubic interpolation method.

Now, what it does whenever a function has been given to us, if the function is differentiable. Then we can proceed further, even if it is not differentiable then also we can use a quadratic interpolation technique, what it does actually I will start my lecture with the quadratic interpolation method.

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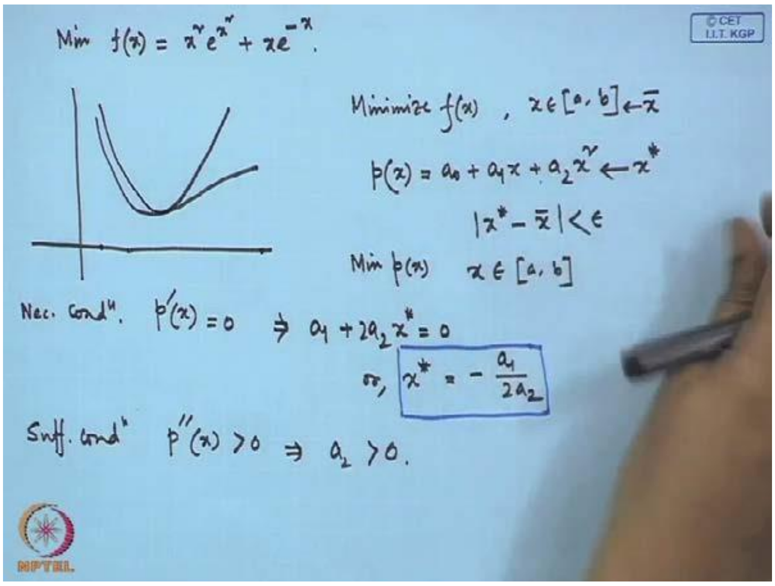

Now, if the given function for example, there is a function f x is equal to say x square e to the power x square plus x into e to the power minus x, some range of x is given to us. Now, if I want to minimize this function, then the function will be differentiable if it is differentiable, now sometimes it happen that it is not differentiable. Now, we can just approximate this function with the quadratic polynomial that is the idea for example, if this is the function for us graph of this function.

Now, the function assumption must be there for applying the quadratic interpolation, and the cubic interpolation method that the function must be unimodel in the given range. Now, if this is the range for the function, then we can just approximate this function with a second degree polynomial say parabola alright, and with it now if I just approximate with this quadratic function, it is very easy to handle the quadratic function that is why we will go for the minimum of this quadratic function.

And we will say, that is the approximate solution of the given non-linear programming problem, that is the idea for quadratic interpolation problem. That us why in general let us consider we are having a function minimize f x and the range for x is given as from a to b. As I said that I will approximate this function with a quadratic polynomial say p x, then it is a second degree polynomial a 0 x plus a 1 x plus a 2 x square, instead of minimizing f x we will minimize p x.

And we will check, if x bar is the minimum value for this optimal solution for this optimization problem. And if x star is the optimal solution for this polynomial, then we will do the iteration we will first find out x star, we will check the condition where the x star is closer to x bar or not. If it is very closer for mathematically we can say x star minus x bar lesser than epsilon, where epsilon is a very small pre assigned value alright.

Now, if this says, so then we will minimize p x and subject to the condition again x belongs to a, b we will go for the minimum value. And how to get the minimum wise as we know the necessary condition would be, first order derivative of p with respect to x must be is equal to 0 that is why, we get from the given polynomial this is equal to a 1 plus a 2 a 2 x and at optimal point at x star this must be is equal to 0. Thus we get the condition that x bar must be is equal to minus a 1 by 2 a 2, this is the necessary condition for us.

And what is the sufficient condition that is p double dot x must be greater than 0, as we get the we want to get the minimum of p x, if this is greater than equal to 0 certainly a 2

must be greater than 0 that is the condition for optimality of p x. Now, our task is to determine the values for a naught, a 1 and a 2 because, these are the coefficient of the corresponding p x. Now, for finding out the values for a 0, a 1, a 2 we need to have since 3 unknowns are there, we need to have 3 equations for this case.

Now, for getting 3 information we can consider 3 points in the interval a to b, say x 1, x 2, x 3 and we will say that f x is equal to p x at this 3 points, then we will get 3 equations. And with 3 unknowns the coefficients of the polynomial a 0, a 1 and a 2 and we can solve it, otherwise we can have information in other way also, functional value at 2 points say x , and x 2. We know the f x value, we will equate f x with p x at this 2 points x , and x 2, and after that there is another information that is the first order derivative of f x is given in one of these 2 points that is why f dot x , or f dot x 2 is known to us.

In this case also we are having 3 equations, and we will have 3 unknowns a naught, a 1 and a 2 that is why we can solve that polynomial rather we will get that polynomial. And we will check the minimum value of that polynomial close to the minimum value of f x or not, how to check I will tell you in the next, but first I will just tell how to find out the value for a 0, a 1 and a 2.

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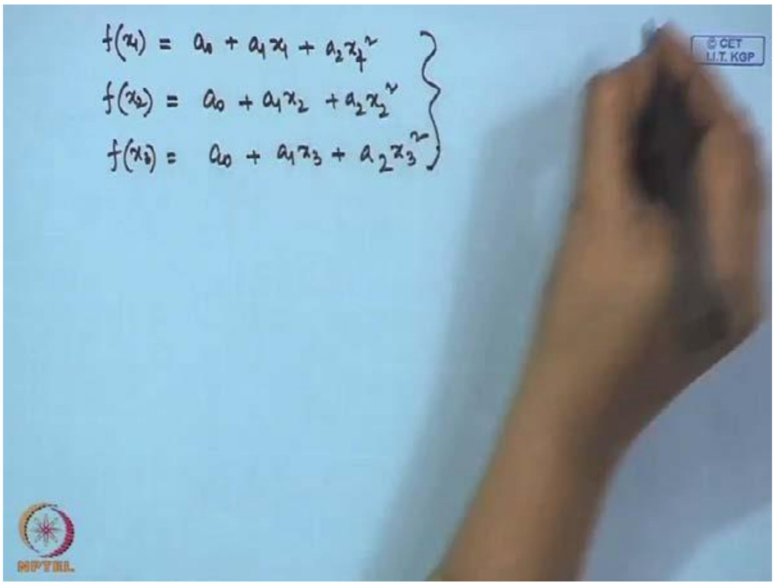

As I said that functional value at this 3 points x 1 is equal to p x; that means, it is equal to a 0 plus a 1 x 1 plus a 2 x 2 square. There is another point in the given interval a b that is x 2 we have a 0 plus a 1 x 2 plus a 2 x 2 square this is x 1, f x 3 is equal to a 0 plus a 1 x 3 plus a 2 x 3 square. Now, we are having 3 equations and we these are the functional values this these are constant for us that is why solving this 3 equations we will get the coefficients a 0, a 1 and a 2 like this.

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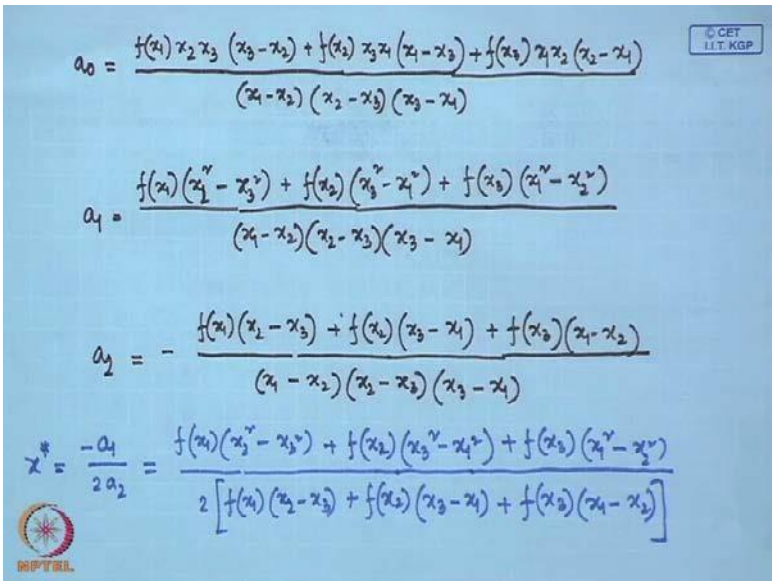
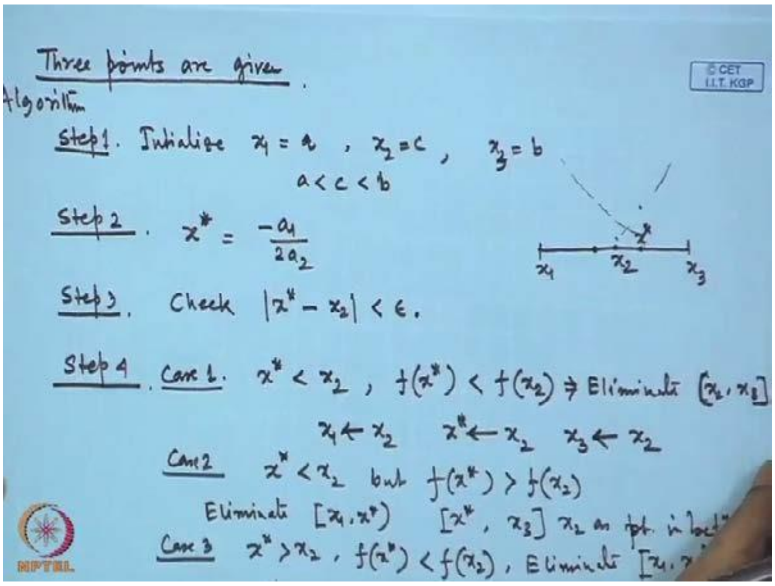


Just after some simplification we get a 0 is equal to f x 1 x 2 x 3 into x 3 minus x 2 f x

plus f x 2 x 3 x 1 into x 1 minus x 3 plus f x 3 x 1 x 2 x 2 minus x 1 divided by x 1 minus x 2 into x 2 minus x 3 into x 3 minus x 1. And in this way we will get the values for a 0, a 1, and a 2, once we get the value for a 0, a 1 and a 2; that means, we are getting the corresponding second degree polynomial. And we can construct p x, now we will go for the optimal value for this p x.

Now, the optimal value for this p x a basic condition is that x star that optimal value must be equal to minus a 1 by 2 a to just I got it from the necessary condition that is why we will just substitute the values for a 1 and a 2, what we got in the previous case here. And we will get the value for x star, and we will check in the next whether x star is the optimal value or not. If it is optimal we will declare this is the approximate optimal value of f x if is not, so then we have to proceed further that is why, let me write down the algorithm for this how to get the solution through the quadratic interpolation method. (Refer Slide Time: 09:03)



When 3 points are given to us, now algorithm tells us the algorithm we can construct in this way because, just now we I have gone through the process. The step 1 would be initialize x 1 is equal to a some given value, and x 2 is equal to b, x 3 is equal to b I can take a middle value here that is c, where c is in between a and b. That means, we are having 3 points here a, b and c, if c is not given to us we can take the middle value of a and b, if x c is given to us then we will get c accordingly.

Now, selection of c is very important because, if c is for from the optimal value, then we need to do more number of iterations. If the c slightly selected, then we will get the optimality very quickly we will move to the next step, we will calculate the value for x star as minus a 1 by 2 a 2 alright, and how I will get a 1 and a 2 I have just showed it to you. Now, step 3 we will check the condition whether x star is close to c or not, if it is close to c; that means, the difference between x star and x 2 is very small that is epsilon, then we will stop.

Otherwise we will proceed to the next step, step 4 this is the interval for us, this is x 1, this is x 3, and the middle value is x 2. Now, we are getting x star, x star can be this side of x 2 x star can be this side of x 2 as well, if x star is in the left hand side; that means, we are considering case 1 that is x star is in the left part of x 2, where f x star is lesser than f x 2. Then what is happening this, is a process lancer's technique that is why our initial interval of uncertainity is given x 1 to x 3.

We are running the quadratic interpolation technique, and in each iteration the interval size will be reduced; that means, we will eliminate a portion of the interval in each step. So, that we will get at the optimal step the interval size is very small, then only it is accepted to us, now x star is f x star is lesser than f x 2; that means, it is very clear that, the optimal value cannot lie from x 2 to x 3. As I assume that function is unimodel in the interval x 1 to x 3 if this is, so then we will eliminate x 2 and x 3, x 2 to x 3.

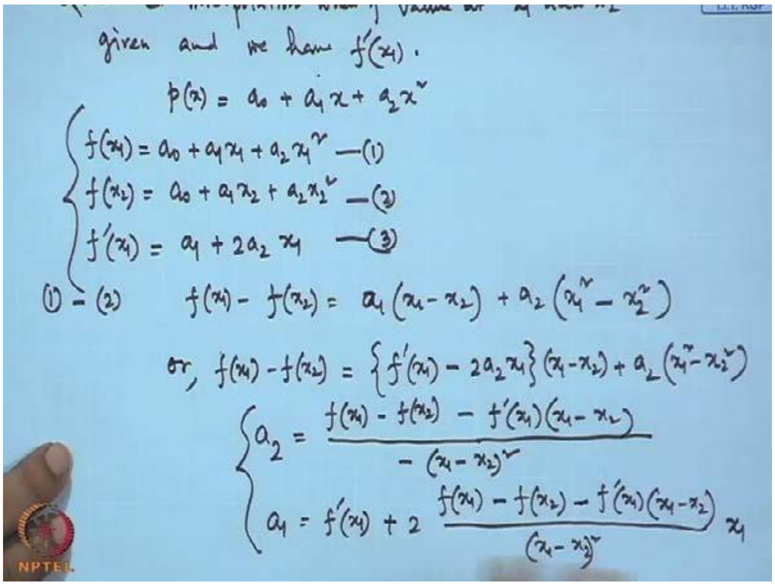
And we will declare the new x 1 as x 1 new x 2 as x star, and we will consider x 3 as x 2 alright. Then we will get another interval x 1 x star x 3 that is; that means, we will get up to this, and we will run the we again we will find out the x star value, if x star is closer to the middle value of that not the middle value, middle point of that interval. Then it is accepted otherwise rejected this is one of that case I have considered that x star is in the left part of x 2, and functional value of x star is lesser than f x 2.

If the other case arises that x star is lesser than x 2, but f x star is greater than f x 2, since we are going for the minimization of functional value. That means, that f x star value is higher than f x 2 value, function is coming in this way in that case; that means, the minimum value cannot lie within this interval that is x 1 to x star. If I consider this as x star thus what we will do, we will eliminate x 1 to x star, and we will declare the new interval from x star to x 3 considering x 2 as the meet at the point in between clay.

The other case may arise as well that is x star is in the right hand side of x 2, but f x star is lesser than f x 2; that means, function is x star is here now, and functional value of x f x star is lesser than f x 2. That means, function is coming down in this way, then we can eliminate up to this there cannot lie any minimum value because, f x 2 value is higher than f x star value. That is why we will eliminate x 1 to x 2 because, minimum cannot lie there.

And we will again apply the technique that is we will calculate with the new points a, b, c, we will calculate x star. We will check whether this is very close to the meet middle value of these interval x 1 to x 2 to rather this is x 2, x 2 to x 3 and we will proceed. Case 4, once I will apply this technique for the problem, then it is much more clear to you now if x star is greater than x 2, but f x star is greater than f x 2, it means that f x star value is higher than the f x 2 value.

That means, the function is coming in this way in this case, then the minimum cannot lie from x star to x 3 that is why we will just eliminate this interval in the next alright minimum must lie from x 1 to x star in this case. Then again we will redefine a, b, c for each of the cases when whatever will occur, we will see the values for a, b, c we will calculate x star, and we will check whether this is optimal or not this is the quadratic interpolation technique when 3 in 3 pors the functional values are given to us alright. We can have another case as well for the quadratic interpolation method. (Refer Slide Time: 17:06)



That is the method when only 2 point are given to us, functional values at x 1 and x 2 given, and we have the first order derivative of the function and the value at x 1 is also given to us. Then in this situation how to apply quadratic interpolation technique because, the previous case it was that 3 point values are given, 3 points are given, here 2

points and one first order derivative of f, if it is given again we will apply the same technique we will approximate the function with a polynomial p x.

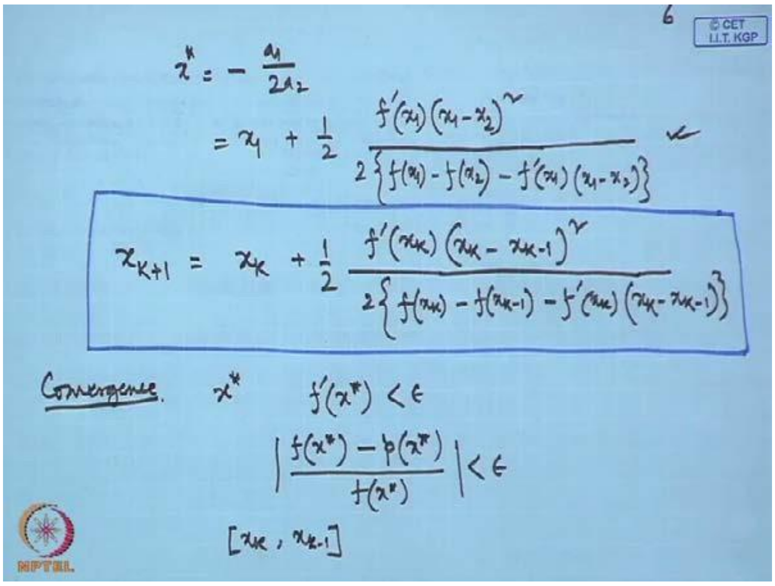
And the p x would be equal to a 0 plus a 1 x plus a 2 x square, we need to find out the coefficients of a 0, a 1 and a 2. So, that we can approximate the polynomial, we can approximate the function f x with the polynomial p x, now when these are given to us then we can have 3 equations like this f x 1 is equal to a 0 plus a 1 x 1 plus a 2 x 1 square, we are having f x 2, f x 2 is equal to a 0 plus a 1 x 2 plus a 2 x 2 square. We are having the functional value x 1 f prime x 1.

Certainly this is equal to a 1 plus 2 a 2 x 1 3 equations again 3 unknowns for us, we need to find out the values for a 0, a 1 and a 2 with this 3 equations 1, 2 and 3. Just I have give you some simplified value how to proceed for that, let us go for 1 minus 2 then we are getting f x 1 minus f x 2 is equal to a 1 into x 1 minus x 2 plus a 2 into x 1 square minus x 2 square.

Now, we will substitute the value in the third equation, then what we get in the third equation from the third equation we will substitute the value for a 1 then we will get f x 1

minus f x 2 is equal to in place of a 1 we will replace f prime x 1 minus 2 a 2 x 1 and the rest thing I will just keep as it is, then this is the equation with only one unknown a 2, from here we will get the value for a 2 as f x 1 minus f x 2 minus f prime x 1 x 1 minus x 2 divided by minus x 1 minus x 2 square.

Now, we can have a 1 as well by substituting the value for a 2, we will get a 1 as f prime x 1 plus 2 f x 1 minus f x 2 minus f prime x 1, x 1 minus x 2 divided by x 1 minus x 2 whole square and here x 1. Once we are getting a 1 and a 2 we may not calculate a naught because, as we have seen for from the necessary condition that p prime x must be is equal to 0, and it gives me the stationary point x star as optimal value, and this is equal to minus a 1 by 2 a 2 that is why for calculation of the optimal value we may not calculate a naught with this two values we can proceed to the next. (Refer slide Time: 21:29)



That is x star is equal to minus a 1 by 2 a 2, and this gives me the value as x 1 plus half f prime x 1 x 1 minus x 2 square divided by 2 f x 1 minus f x 2 minus f prime x 1 into x 1

minus x 2 this is the value for the optimal point. Now, we can develop the iteration process with these from this equation, by substituting x star as the next approximation when x k and x k minus 1 values are given to us.

And this can be written as and by substituting this as x k and x k minus 1 we will get x k plus half f prime x k, x k minus x k minus 1 square divided by 2 f x k minus f x k minus 1 minus f prime x k x k minus x k minus 1. This is the iteration formula for us that is why when x k and x k minus 1 is given both the values are given to us, we will calculate the value for x k plus 1. And we will go for the check whether the convergence of the sequence of values is there or not, for checking the convergence there are few processes.

First thing is that, if x star is the optimal value for us it could be minimum, it could be maximum. Then at this point the function also has the optimum the minimum or maximum value that is why I can say that f prime x star is very small closer to 0, I cannot say this is equal to 0 because, I am approximating f x with the polynomial p x. That is why it may not happen that this is the exact minimum for f x that is why we can check, whether f prime x is lesser than epsilon or not.

If it is lesser than a very pre assigned small value, then we can accept x star as the optimal solution. There is another check also there, this is available in the literature we can also go for the f x star minus p x star divided by f x star. If this value is lesser than epsilon that also could be a check, and other check as I said that if the interval that is x k and x k minus 1 this interval is very small, then also we can stop our iteration process.

These are the all alternative processes for checking the convergence of the sequence of values of x k plus 1, and rather that will lead us to the optimal solution for the given function f x.

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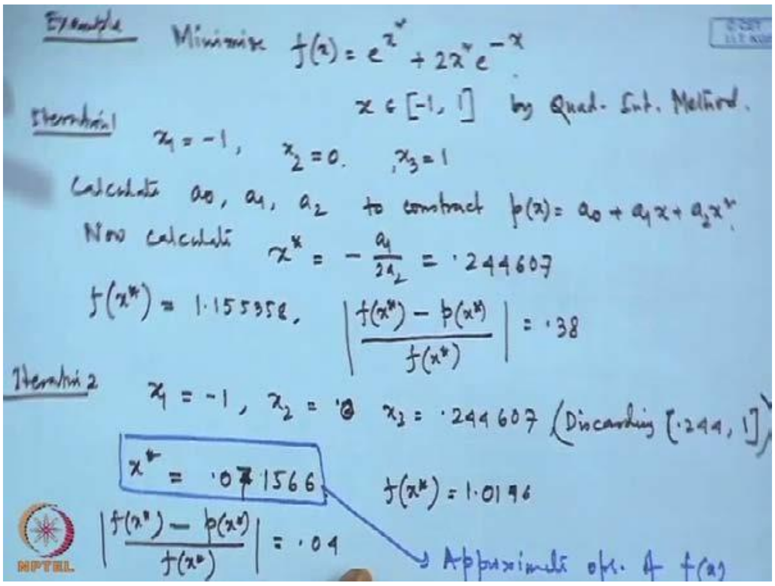

Now, let me apply this one for a problem given like this example, where we need to minimize f x is equal to e x square plus 2 x square e to the power minus x. And the interval has been given that x is in between minus 1 to 1, as we see the function involves a exponential function. We can go for the other techniques as well for getting the optimal solution, but if we apply the quadratic interpolation technique it has been seen that, quadratic interpolation technique is much more efficient then than the Fibonacci golden search technique as I discussed before.

And thus quadratic interpolation method convergence is very quick, very faster than other methods. Then we can apply the quadratic, and I will discuss in the next the cubic interpolation, and we will see cubic interpolation technique is much more efficient than other line search techniques like Fibonacci golden section an others and the quadratic interpolation technique as well.

Now, if we apply the quadratic interpolation technique, then as we see in the first iteration, we are having the value as x 1 as minus 1, x 3 is equal to 1. And as I said if 3 points are not given to us, if no other information is given then we can consider the middle value as x 2 point that is why we having x 1 minus 1 x 2 equal to 0 and x 3 is equal to 1. Then we can calculate the values for a 0, a 1 and a 2 to construct p x, see how to get this values as I just showed you, we will calculate a 0, a 1 and a 2 with this formula.

Because, we are we know x 1, we know x 2, we know f x 1 f x 2 f x 3 as well, we will just substitute the values here, and we will get a 0, a 1 and a 2. From there we will calculate x star. This is equal to minus a 1 by 2 a 2 with the formula I have showed, now this value will be 2.244607; that means, in this iteration we are getting the minimum point of the given polynomial p x, with these a 0, a 1, a 2 values is x star. And we can declare this is the approximate value of the minimum value of the function.

But, we need to check whether this is correct or not that is why we will go for the functional value for f x at x star. We see the functional value is 1.155358, and we will check if I just check the condition, as I said I can check the first order derivative of function at point x star it if it is very small. Then we can stop our iteration or other check as I said we will just check the value the difference between x star and x 2, we see the difference is 0.244 that is very high we cannot stop.

But, we can check other way as well, as I was discussing that is the this is the value, and this value is coming as 0.38. This is also quite high that is why we should not stop here, we cannot declare x star though this is the minimum value of the polynomial p x, but we cannot declare x star, as the minimum value for f x that is e to the power x square plus 2 x square e to the power minus x, we will proceed to the next. How I will proceed to the next iteration.

Iteration 2; that means, the interval we are having minus 1, 0 and minus 1 and 1, this interval will be reduced in the next iteration. And this will be reduced according to the unimodality property, as I said function is unimodel in the given interval that is why we will check the functional value at minus 1, functional value at 0, functional value at 1. And also functional value at this point, and we see the functional value at 1 will be higher than the functional value of x star.

That means, minimum cannot lie within that and functional value is decreasing, again that is why we will consider the next interval as x 1 is equal to minus 1, x 2 is equal to 0 and x 3 is equal to 0.244607. That means, we are discarding 0.244 to 1 because, with unimodality property, we can check the minimum cannot live here. Again we will go for the calculation for a 0, a 1 and a 2 and we will get x star as well here, x star will come as in this situation this is 0.071566.

And we will check the functional value at this point as, well we will see the functional value is 1.0146. And we will see this functional value is very close to the functional value at 0, functional value at 0 is 1 it is very close that is why, we can either go for the check as

I said. The first order derivative of f of f at point x star, we will see the value will be very less, and otherwise we can calculate this value as well f x star minus p x star divided by f x star, and this value is coming as 0.04.

This can be considered as a very small value, if we are happy with this value we can stop our iteration. And we can declare that this is the optimal solution for the approximate optimal solution for the given function, but if you are not happy with this value we can proceed in the next again how. Again we will just reduce the interval from minus 1 to this point with the unimodality property, and accordingly we can proceed in the next and we will say, we will see that we will get better minimum, better x star.

As many iterations we will do, we will get better iterations further, but at some point it will converge there we need to stop at least. Since I am doing it manually I am stopping here with the acceptance that this value is acceptable to me, now this is the way we can use the quadratic interpolation method, for solving any function given and function can be complicated in nature also. Then also we can apply the technique very well that is all about the quadratic interpolation method.

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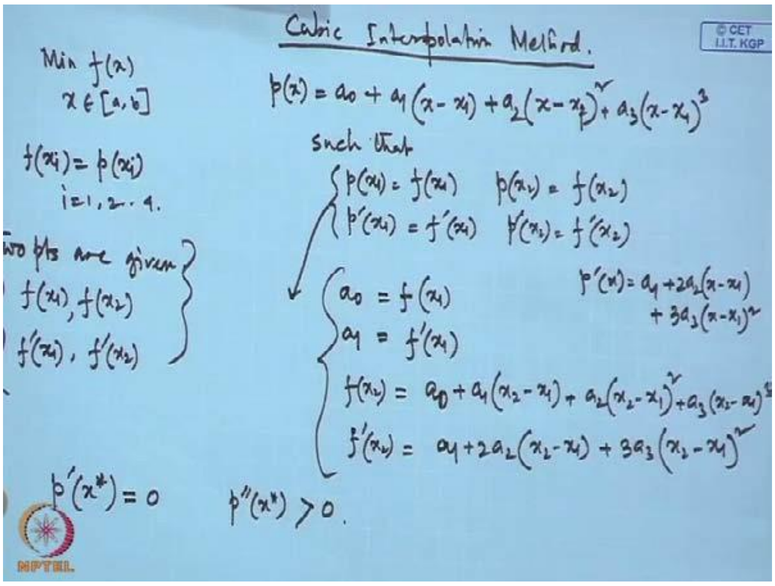

Let me go to the next that is the cubic interpolation technique that is the extension of the quadratic interpolation technique. But, cubic interpolation method is popular and much more better method than other line search technique, but one disadvantage of this method is that, the calculation is very long. Because, as you see we are going for the cubic interpolation method; that means, we are instead of approximating the function with the second degree polynomial.

We will approximate the function with the third degree polynomial that is cubic function, and since this is the third degree polynomial instead of having in the quadratic, we had the 3 unknowns. Here we are having 4 unknowns that is why the information we need 4 information to calculate the value let me go to the process further, the cubic interpolation technique. We are having the function minimize f x x belongs to a, b we are approximating the function f x with the polynomial p x, and this polynomial is of third degree polynomial.

Now, for doing that we are having 4 unknowns that is why we need 4 equations, how to get 4 equations, either the functional values at 4 points should be given to us. And we will equate that and the functional value of the polynomial, we will equate both together that is either we will calculate f x i is equal to p x i, for i is equal to 1 to 4. And we can get the values for a 0, a 1, a 2 and a 3, otherwise we can have functional values at 2 points and derivative value at 2 points and the same 2 points.

That means, we will have 4 equations again, and we are having 4 unknowns that also quite possible. And otherwise we can have the 3 points are given, and 1 derivative value also given it is it could be happen also, there also we will have 4 equations and 4

unknowns and we can solve it. Now, I will just consider one case because, other case calculations are almost similar to the this case that is why we will consider the case where 3 no 2 points are given.

That means, we are having f x 1 f x 2, and we have the derivative of f at x 1 we have derivative of x 2 at x 2 alright. These are all given to us, and with this information we will proceed to approximate the polynomial, we can consider the polynomial in this way a 0 plus a 1 into x minus x 1 in different just little deviation I have taken from the quadratic interpolation technique. Again we can just approximate this polynomial as a 0 plus a 1 x plus a 2 x square plus a 3 x cube that also quite possible.

But, here I am taking another in another way, so that I will get a nice and quick result from here cube, in such a way that p x 1 is equal to f x 1 p x 2 is equal to f x 2 p prime x 1 is equal to f prime x 1, and p prime x 2 is equal to f prime x 2. Then we will have 4 equations and 4 unknowns, then from this information what we get that a 0 must be is equal to f x 1 alright.

From the second p prime x 1 is equal to f prime x 1, p prime x 1 means a 0 plus a 1 x 1 alright now sorry a 0 no only a 1 because, p prime x is equal to a 1 plus 2 a 2 x minus x 1 plus 3 a 3 x minus x 1 square that is why from here, we are getting at point x 1 if p prime x 1 equal to f prime x 1, than we are getting a one is equal to f prime x 1 alright.

Now, what about p x 2, p x 2 gives me the p x 3 equal to f x 2 gives me the equation f x 2 is equal to a 0 plus a 1 x 2 minus x 1 plus a 2 x 2 minus x 1 square plus a 3 x 2 minus x 1 cube. Here you see we are we know the value for a 0, we know the value for a 1 that is why this will be equation with only 2 unknowns, a 2 and a 3 because, x 1 x 2 f x 2 everything is given a 0, a 1 can be calculated very easily from the functional values at x 1, and derivative value of that function at x 1.

And from the other condition that is p prime x 2 is equal to f prime x 2, from here we will get this value f prime x 2 is given to us that is why will equate these with a 1 plus 2 a 2 x 2 minus x 1 plus 3 a 3 x 2 minus x 1 square. And with this 4 equations and 4

unknowns we can get the solution very easily, and a 0 and a 1, a 2 and this are the equations with a 2 and a 3 only, that is why again we will declare the necessary condition for getting optimal is p x star is equal to 0. Not only that p double prime x star must be for minimization greater than 0.

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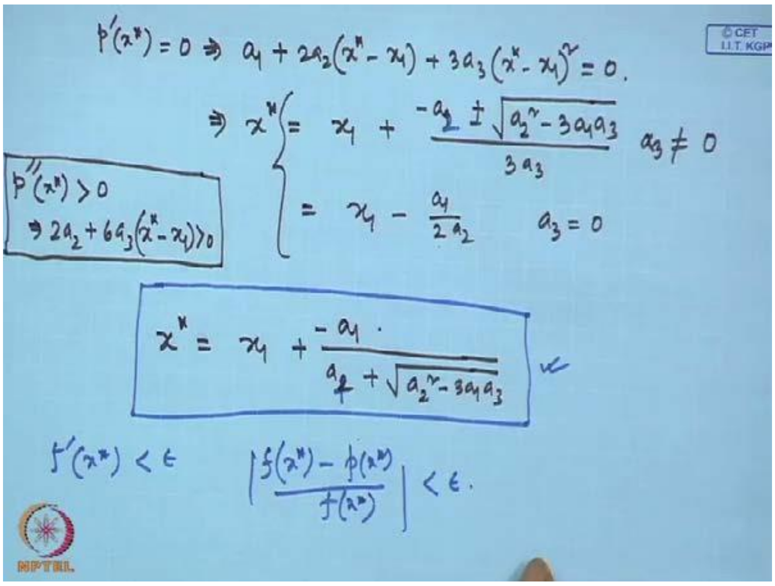

And if we equate p prime x star is greater the is equal to 0 we get the value for x star as; that means a 1 plus 2 a 2 x star minus x 1 plus 3 a 3 x star minus x 1 square is equal to 0 this is the equation for us. From here we get the value for x star as x 1 plus minus a 1 plus minus root over a 2 square minus 3 a 1 a 3 divided by 3 a 3 because, this is the equation of degree 2, we can use this and we will get the value for x star. And we are having another condition as well p prime x star is greater than 0.

That means, we are getting that 2 a 2 plus 6 a 3 x star minus x 1 greater than 0, this is another condition is given for minimum, this is the sufficient condition. Now, when a 3

is equal to 0 here, we get the value for x star as x 1 minus a 1 by 2 a 2, when a 3 not equal to 0 and we are getting a 3 equal to 0. Thus if we calculate the values for a 1, a 2 and a 3 from the given conditions as I showed before.

Then we will get the value for x star is equal to this, either this or that instead of considering both together, we can just simplify the case. And we can declare by multiplying both side the numerator and denominator with certain value, we will get x 1 plus a 1 divided by a 2 plus a 2 square minus 3 a 1 a 3 and this can be considered as general equation, you see we have considered only the positive values, positive value of x star.

Why because, sufficient condition tells us that if we consider negative value then it will not satisfy thus, we will only consider the plus value here, and we are multiplying numerator and denominator with a 1 plus this and we will get the solution for the given equation.

Now, if this is, so you see there is a nice fact here if we consider this function even a 3 is equal to 0, it leads to this value that is x 1 minus a 1 by a 2 this is the value, here it should be a 2 not a 1 alright. Now, this is the way we can calculate the optimal value for the cubic interpolation, and the same procedure if we are satisfied then we can proceed to the we will proceed, if you are not satisfied we will proceed to the next iteration after checking the convergence.

Convergence checking is again the same that we will check whether f x star value is small or not, at either we can go for this or we can calculate f x star minus p x star divided by f x star. Whether this is lesser than a very small value or not, and we can proceed further in this way.

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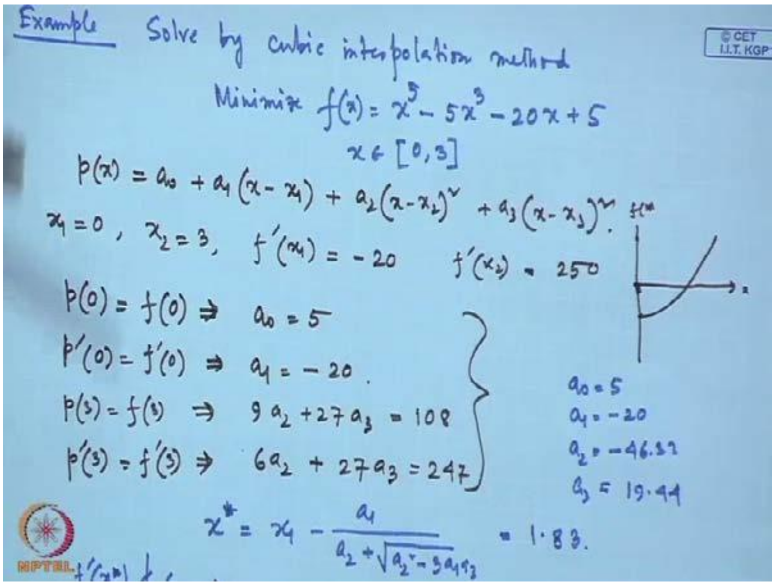


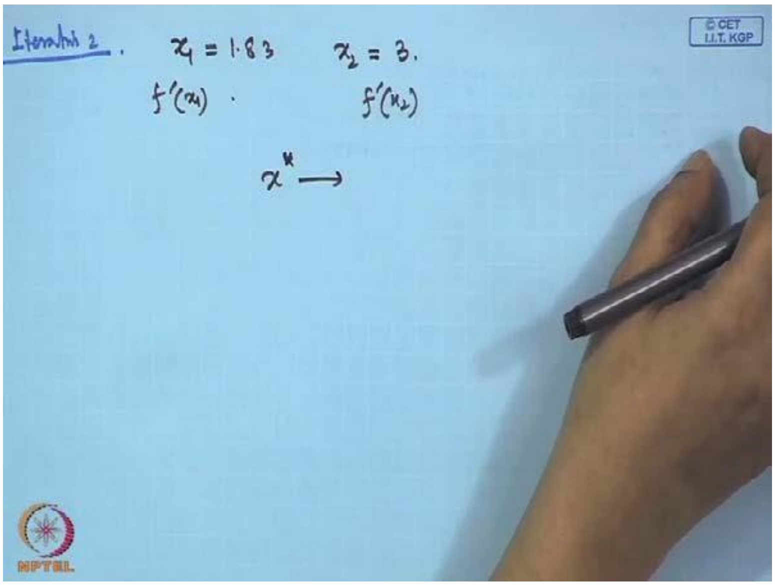
Let me take one example for this, the problem is minimize f x we are having a another polynomial of degree 5, and initial interval of uncertainity has been given that from 0 to 3. And the cubic interpolation technique, we are we will apply means we will approximate this fifth degree polynomial with a lesser order polynomial that is degree 3. Thus we will approximate p x with f x with p x, as a 0 plus a 1 into x minus x 1 plus a 2 x minus x 2 square plus a 3 x minus x 3 square.

Now, for this we are having information regarding we know only x 1 equal to 0 x 2 equal to 3, we can have f prime x 1 value that would be is equal to minus 20, and f prime x 2 value that would be is equal to 250 alright. Then the function looks like this, and we have assumed that function is unimodel this is x, this is f x; that means, function is coming it is negative, and negative to positive it goes in this way alright, and minimum may lie it would be this way as well alright.

If this is, so then we will calculate the values one after another, how we will go for p 0 is equal to f 0, p 0 equal to f 0 gives me the value for a naught is equal to 5, p prime 0 is equal to f prime 0, this gives me the value for a 1 is equal to minus 20, p 3 is equal to f 3 this gives the equation 9 a 2 plus 27 a 3 is equal to 108 by substituting the value of a 0 and a 1. And we are having another equation p prime 3 is equal to f prime 3, then it will give me another equation 6 a 2 plus 27 a 3 is equal to 247 alright.

Now, we are having 2 equations and 2 unknowns, and from here we will get the values for a 0 as 5 a 1 as minus 20, a 2 as minus 46 dot 33 and a 3 is equal to 19.44. And we will calculate x star here, x 1 minus a 1 divided by a 2 plus root over a 2 square minus 3

a 1 a 3, and this value is coming as 1.83. And if this is the value and we will go for the functional value, and we will see that functional value is a not that is a first order derivative of the function is not small enough that is why we will go to the next iteration. (Refer Slide Time: 47:45)



How to get the next iteration, as we see that functional value is at 1.83 functional value is less that is why the next interval we will consider as 1.83 and x 2 is equal to 3. And we will calculate the again we will go for f prime x 1 and f prime x 2, and we can calculate x star from here. And if this optimal solution is acceptable, then it is accepted otherwise we will go to the next iteration further for getting the solution of the problem.

And if this is the way we can solve the quadratic interpolation the minimization or maximization of any non-linear function, with quadratic interpolation or with cubic interpolation technique.

Thank you for today.