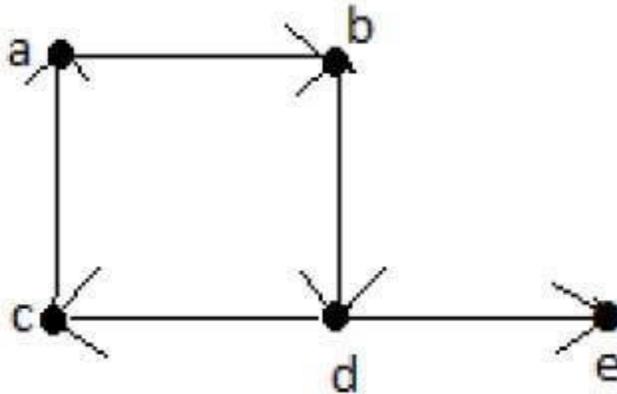


Paths and Circuits

Basic Definitions

A. Euler's Path:

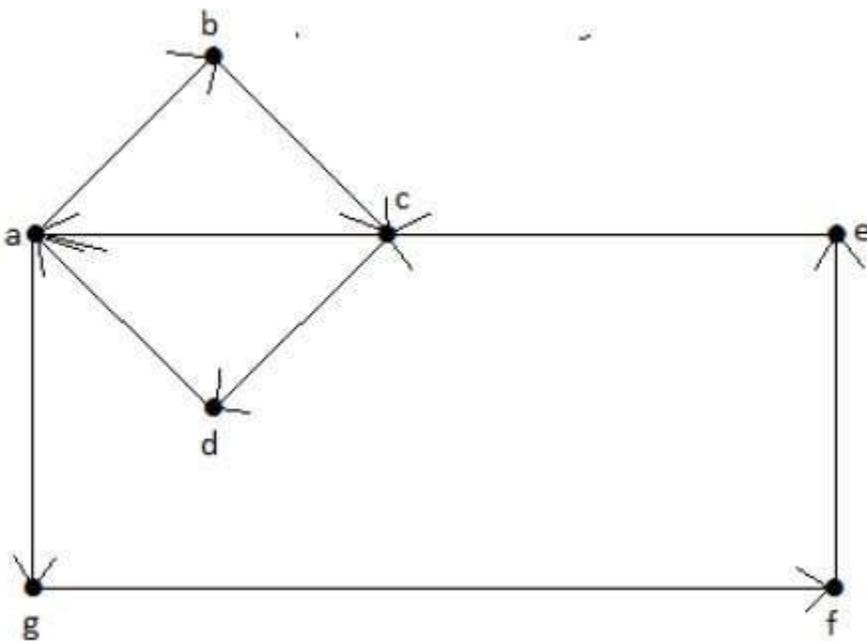
- An Euler's path contains each edge of Graph 'G' exactly once and each vertex of 'G' at least once.
- A connected graph G is said to be traversable if it contains an Euler's path.
- Example



Euler's Path = d-c-a-b-d-e

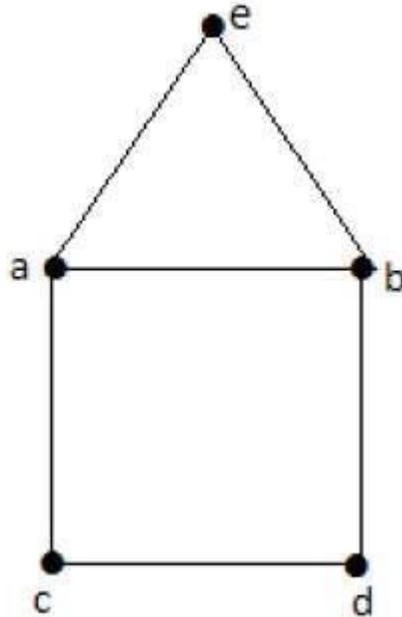
B. Euler's Circuit

- In an Euler's path, if the starting vertex is same as its ending vertex, then it is called an Euler's circuit
- **Example:** In the following figure, **Euler's Circuit = a-b-c-d-a-g-f-e-c-a.**



Properties of Euler's Graph

1. **Theorem 1:** A connected multigraph with at least two vertices has an Euler Circuit if and only if each of its vertices has even degree.
 - In a connected graph G , if the number of vertices with odd degree = 0, then Euler's circuit exists.
2. **Theorem 2:** A connected graph has a Euler Path but not a Euler Circuit if and only if it has exactly two vertices of odd degrees.
 - **Example**



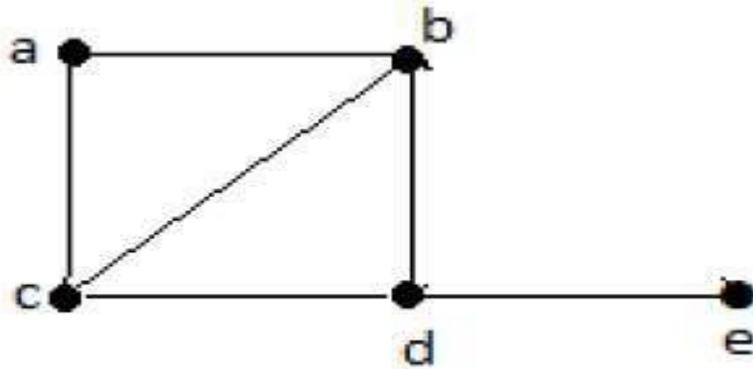
Euler's Path – b-e-a-b-d-c-a, is not an Euler's circuit, but it is an Euler's path. Clearly it has exactly 2 odd degree vertices.

3. If a graph has an Euler Circuit, it cannot have an Euler Path, and vice-versa

NOTE: IN EULER PATHS/CIRCUITS, THE AIM IS TO FIND PATHS/CIRCUITS THAT INCLUDE EVERY EDGE OF THE GRAPH ONCE AND ONLY ONCE

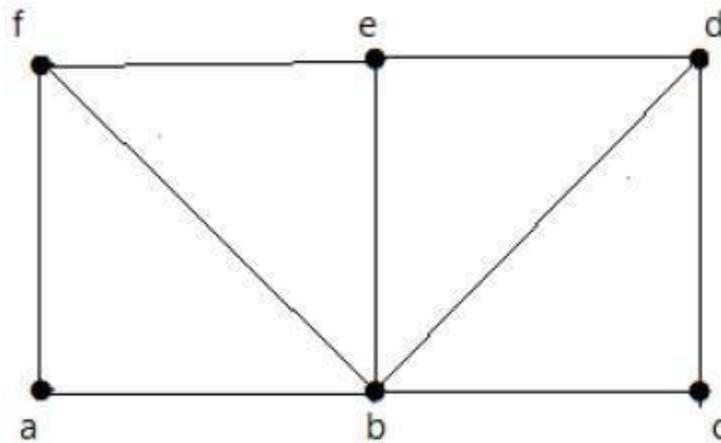
c. Hamilton Path:

- An undirected connected graph is said to have a Hamiltonian Path if it contains each vertex of Graph G exactly once.
- **Example:** In the following figure, **Hamiltonian Path**– e-d-b-a-c.



D. Hamilton Circuit (Cycle): In an undirected connected graph, the Hamiltonian cycle or circuit is a circuit that includes each vertex of the graph exactly once.

- **Example**



This graph has a Hamiltonian cycle as well as Hamiltonian path passing through all the vertices.

Properties of Hamilton's Graph

1. If a graph has a Hamiltonian circuit, then it automatically has a Hamiltonian path.
2. A graph can have a Hamiltonian Path, but not necessarily a Hamiltonian Circuit.

NOTE: IN HAMILTONIAN PATHS/CIRCUITS, THE AIM IS TO FIND PATHS/CIRCUITS THAT INCLUDE EVERY VERTEX OF THE GRAPH ONCE AND ONLY ONCE