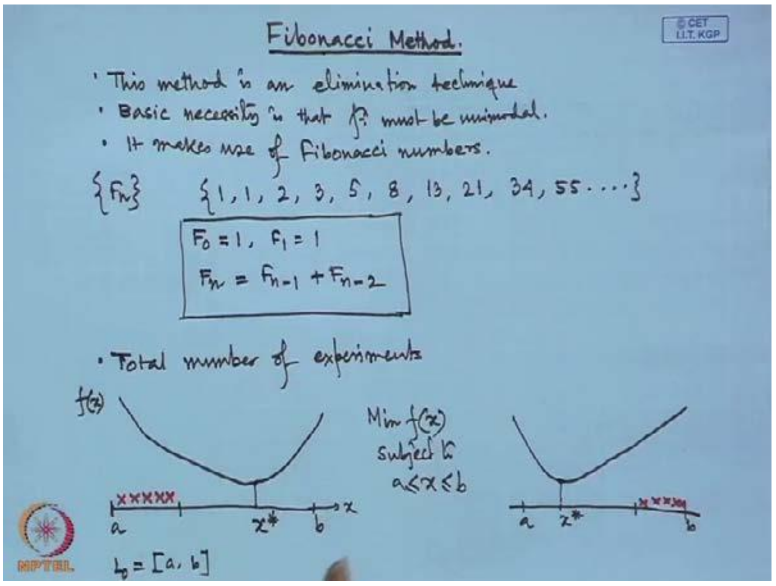
NOTE: The following lecture is a transcript of the video lecture by Prof. Debjani Chakraborty, Department of Mathematics , Indian Institute of Technology Kharagpur.

Fibonacci Method

Today's topic is Fibonacci method. This method is again another elimination technique, for solving single dimensional one variable non-linear optimization problem. The basic necessity for applying this method is that, the function must be unimodal in the initial interval of uncertainty. Now, one thing must be said that this Fibonacci method, the beauty of this method is that it makes use the Fibonacci numbers in the sequence. That is, if there are certain limitations of this method as well. Now, let me just first tell what are the basic necessities and what are the limitations for the Fibonacci method. (Refer Slide Time: 01:01)



First of all, this method is an elimination technique. Elimination technique, that is another name can be said as the interval reduction method, for solving for getting optimal for non-linear optimization problem. And the basic necessity for applying this method is that, as I said this is the basic necessity is that the function must be unimodal. Function means the function which I am going to optimize, must be unimodal in the given range.

And is its makes use of Fibonacci numbers. As we know, there is a there is a well known sequence of Fibonacci numbers or Fibonacci people as saying in that name as well. If we consider F n is the sequence, that is the Fibonacci numbers and we must be knowing that this is the this is the sequence we are getting 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 in this way. If we just look at the sequence, we see it follows certain rules. The rule is that, if I consider F naught is the first number, F 1 is the second, F 2 is the third in that way and we see that F 0 is all over is equal to 1, F 1 is equal to 1. And if we consider other number, that would be sum of the previous 2 numbers. That is why F n must be is equal to F n minus 1 plus F n minus 2. That is the beauty of the Fibonacci number and we will use these numbers in the Fibonacci method.

And there is one good thing for this method is that, we can we have to define, we have to say the total number of experiments before initiation. Total number of experiments or the total number of approximation we are making for optimal solution, that has to be said beforehand. And if this is so, now let us say this is my, this is my function, this is the given interval from a to b and this is the function.

As I said, function must be unimodal, say we want to minimize the function, minimize f x and this is the range x. Then we are having the function minimize f x subject to x in between a to b. If we look at the graph, it is very clear that it has the minimum at this point. Let us say, x star is the minimum for this function. Now, let me see another function. Again also from a to b and this is the function for us. Now we see that function is having minimum at this point, this is my x star, this is the optimal solution for this function.

Now, we are just we are want to implement one iterative processes that is the Fibonacci method for solving this minimization of f x. If we just look at the property of the function, we see that function is having minimum here. That is why, we can just have the reduction, interval reduction process in this way that, in the first iteration we will consider the initial interval. Say, initial interval is a to b and we will say this initial interval of uncertainty as L naught. Now, since we are having minimum here, in the next level we can, we may just discard this part of the interval. So that in the next iteration, we will just consider this interval and in this case, we will just discard may be we may discard this part.

Now, that is why the basic philosophy of the region elimination technique is that similar to Fibonacci method is the as well, in every iteration we reduce a part of the interval, so that we will just we will conclude, we will just complete, we will just conclude our iterative process with a shorter interval, with a small interval where we will declare the optimal is laying in that interval. That is why the process is like that, we will start from initial interval of uncertainty L naught is equal to a to b.

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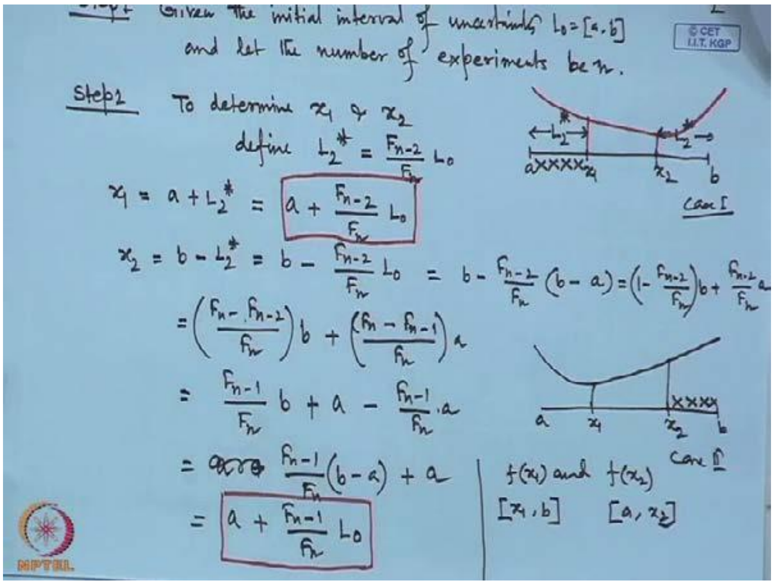

Now, my next process. My step one that is why is that, for Fibonacci method is that given the initial interval of uncertainty, let me just elaborate the iterative process L naught from a to b. Now as I said, this is the number of experiments must be said beforehand. Let we considered, let the number of experiments be n. Once we are specifying the number of experiments, then since we are using the Fibonacci numbers, that is why we are sure that the we will considered the Fibonacci numbers up to F n. If we are having 5 number of experiments, then we will use the Fibonacci numbers F 0, F 1, F 2, F 3, F 4 and F 5 in that way, just I will show the thing with a example in the next. Now just, let us go to the next step, that is step 2. What is my basic?

Basic idea is that, we will reduce a part of the given interval of uncertainty, that is a to b. That is why the process tells us that, we will considered 2 points, 2 experiments here. Say x 1 and another one is x 2, in such a way that there is a specific distance of x 1 and x 2 from both the end of the given interval. In the next step, in the step 2, I will just show how to get the next approximations for the optimal solutions x 1 and x 2. That is why, to determine x 1 and x 2, what we have to do. That is initial 2 experiments, approximations will be defined in this way. We need to define one number, that is L 2 star. The L 2 star would be is equal to F n minus 2 by F n into L 0.

Here n is the number of experiments as we have specified before. That is why if it is n is equal to 5, then L 2 star must be is equal to F 3 divided by F 5 into length of the initial interval of uncertainty, that is L naught. And why we need L 2 star because what we will do, we will consider the interval a to b and we will just select x 1 and x 2 in such a way that, both the experiments x 1 and x 2 these are all L 2 star distance apart from both the ends of the given interval. That is the L 2 star quantity. That is why once our L 2 star is defined in this way, then we can generate 2 experiments, rather 2 approximations for the optimal solution x 1 and x 2.

Therefore x 1 would be is equal to, since it is L 2 star distance apart from a x 1 must be is equal to a plus L 2 star. In other way, we can write a plus L 2 star is equal to a plus F n minus 2 divided by F n L 0. What about x 2? X 2 is again L 2 star distance apart from b, that is why x 2 must be is equal to b minus L 2 star. That is equal to b minus F n minus 2 divided by F n L 0. This is the basic thing, but we can do certain simplification for x 2

here. How we can do so? We know L 0 is equal to that is the length of the initial interval of uncertainty. That is why we can write L naught is equal to b minus a.

Once this is so, then we can write it down as 1 minus F n minus 2 divided by F n into b plus F n minus 2 by F n a. This again is equal to F n minus F n minus 2 divided by F n into b plus F n minus 2 can be written as we know F n is equal to F n minus 1 plus F n minus 2. That is the n th Fibonacci number is the sum 2 previous numbers. That is why F n minus 2 can be written as, F n minus F n minus 1. That is why, we can write it down in this way.

Once this is so, we can write it as F n minus F n minus 2 again F n minus 1 divided by F

n into b plus a because F n by F n is 1. This is equal to F n minus 1 by F n into a. Thus we are getting it as a plus F n minus 1 divided by. Am sorry, a will not be there. F n minus 1 into F n into b minus a plus a. Thus we are getting a plus F n minus 1 by F n L 0. That is why just look at the another form of x 1, x 2. This is my x 1, a plus F n minus 1 by F n L 0 and what about x 2? a plus F n minus 1 by F n L naught. In other way, we can say this is as b minus F n minus 2 by F n L 0. This is this is the, these are the forms of x 1 and x 2 respectively. How to adopt in the example?

I will just do in the next. Now if this is so, what is my next task? If the function is like this, if this is my function, then certainly at point x x 2 the functional value is lesser than at point x 1. If we consider the other way, this is a this is b and there are 2 approximations where x 1 and x 2 are lying between a to b. The function is like this. Certainly f x 1 value, functional value at x 1 must be lesser than functional value at x 2. That is why, in both the cases if we consider the elimination technique, that is very clear in the case one, we will reduce this interval in the next iteration because minimum cannot lie here.

And if we consider, the next case, case two: then we must reduce this interval from x to b. That is why, if we just consider both the things together, what is my next task? My next task is to find the values f x 1 and f x 2 and by considering the unimodality property, we will reduce either a to x 1, then my new interval of uncertainty would be x 1 to b or in the next case my new interval of uncertainty would be from a to x 2. This is the case for both the cases.

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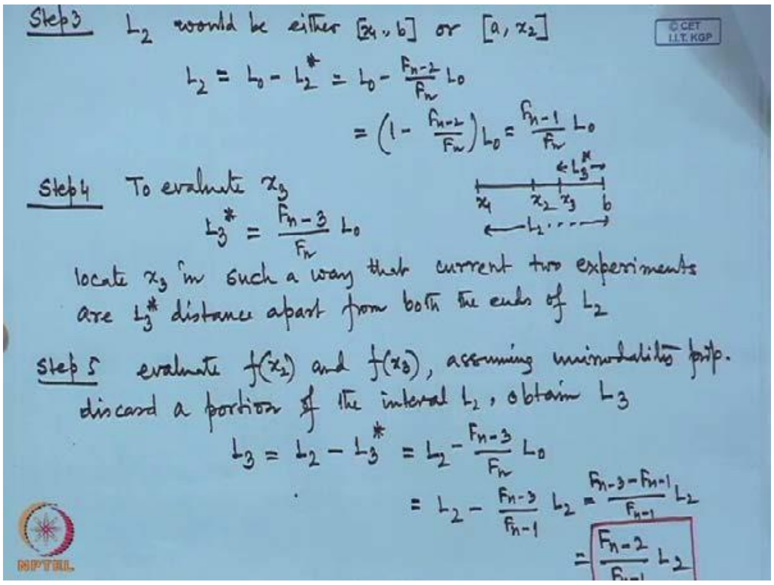

Now, that is why my next step would be, step three would , we will reduce the interval L naught and we will rename the new interval as L 2 as a new interval of uncertainty. That is why my new interval of uncertainty L 2, that would be either as we see from the previous case, either it is x 1 to b or it would be from a to x 2. Accounting to the functional value at x 1 and x 2.

Now, what is the length of L 2 then? What is the next interval of uncertainty let length? Then certainly it would be from x 1 to b or a to x 2 and we will see the nice fact that L 2 will be is equal to L naught minus L 2 star because we are reducing this length. That is L 2 star length, this is length we are we have considered x 2, x 2 from b L 2 star distance apart. That is why next interval of uncertainty that would be is equal to L 2 and this L 2 would be is equal to L naught minus L 2 star. What is L 2 star again?

Let me write it down, L 2 star that would be F n minus 2 by F n L naught and that can be written very nicely as F n minus 2 by F n L naught and the nice fact is that L 2 would be is equal to F n minus 1 by F n L naught. That is my next interval of uncertainty. Now once it is fixed, then we will just find out the next approximation. In the next step, step 4. That is why we will go for next approximation. If it is from x 1 to b, we are having x 2 here. Then we will go to evaluate the next approximation x 3 in the next.

Now to define x 3, again we will have to evaluate L 3 star. L 3 star would be is equal to, as we have seen F L 2 star is equal to F n minus 2 by F n L naught, L 3 star would be is equal to F n minus 3 by F n L naught. Again, locate x 3 in such in a way that, current 2 experiments are L 2 3 distance L 3 distance are apart from both the ends of the interval of uncertainty. Current 2 experiments are L 3 star distance apart from both the ends of L 2. This is my L 2. That is why it would be from that must be position of x 3 in the next case. And if this is so, then again the same process, we will just repeat the process like previous. We will go the step 5. We will evaluate the value, functional value at x 2 and f at x 3 and assuming the unimodality property, we will discard a part of the interval.

Unimodality property, discard a portion of the interval L 2 and obtain the new interval of uncertainty L 3. And what would be the length of L 3? Certainly, since we are discarding this is L 3 star, then the new interval of uncertainty would be L 3 would be is equal to L 2 minus L 3 star. Again if we just write down the thing in detail, that must be is equal to F n minus 3 by F n L naught and this is equal to F n minus 3, that can be written as F n minus 1, L 2 as well. It would be must be is equal to F n minus 3 minus F n minus 1 divided by F n minus 1 L 2 and this would be is equal to F n minus 2 divided by F n minus 1 L 2. And this is the form of L 3.

If you look at the pattern of L 2 and L 3 separately, in L 2 we got F n minus 1 by F n L naught in L 3 we are getting F n minus 2 by F n minus 1 L 2.

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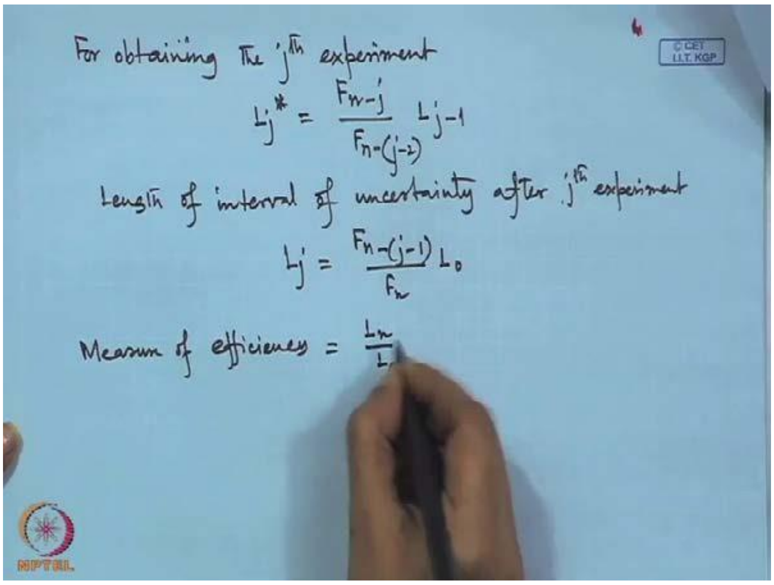
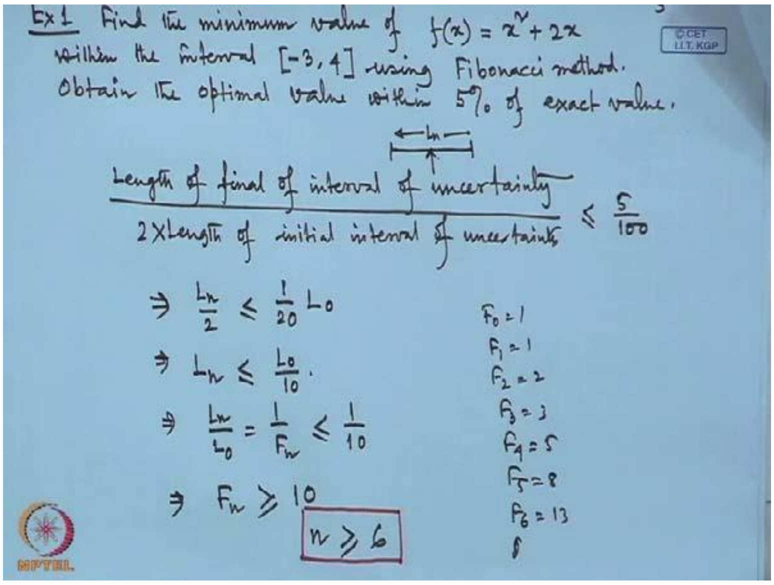


That is why in general, for obtaining the n th experiment. Because as we got x 3, in the similar process we will discard a part of L 3 and we will reach to L 4 and there also we will get another experiment x 5. In that way, we will proceed further and further. How long we will be proceed? We will proceed up to the n th experiment, That is why, if we consider the j th experiment, the formula would be, to obtain the j th experiment point, we need to find out L j star as we got L 2 star and L 3 star and that must be is equal to F n minus j divided by F n minus j minus 2 L j minus 1.

And the corresponding length of uncertainty after the j th experiment, L j would be is equal to, that is the length of the interval of uncertainty would be F n minus j minus 1 divided by F n L 0. If I put L j is equal to 2, we are getting F n minus 1 divided by F n L

0, if j is equal to 3, then we are getting F n minus 2 divided by F n L 0. In that way, we can generator all the points x 1, x 2, x 3 up to x n and we will reduce the interval further and further and at end we will get n th interval of uncertainty and the middle of that interval of uncertainty will be declared as the optimal solution for the given problem. Whatever we I have said, let me just let me just apply the whole process in the in one of the example.

But one thing I must tell here, as I have said for other elimination technique that there is one measure, that is called the measure of efficiency, efficiency of the elimination technique because we have learnt the exhaustive search technique dichotomous search, interval search technique and this is another searching technique that is the Fibonacci method. In the next, we will do the golden section method. For every method, there is a there is a measure for a efficiency. In other another name that is called the reduction ratio, through which we can judge the efficiency of the of the elimination process. And I will discuss more on this, but here just I want to tell you the measure of efficiency or the reduction ratio can be defined as L n minus L 0, where n L n is the length of the interval of uncertainty after n th experiment and L naught is the initial interval of uncertainty. (Refer Slide Time: 24:22)



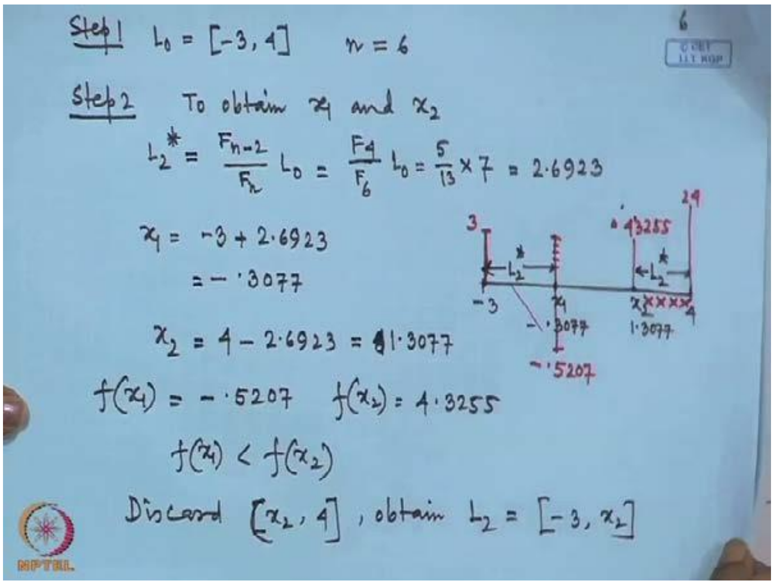
Now this is the thing. Now, let me apply this process for one example. A example is that, find the minimum value of f x is equal to x square plus 2 x, within the interval minus 3 to 4 using the Fibonacci method and obtain the value, optimal value within 5 percent of exact value. Now, this is the problem for us now we want to minimize the function x square plus 2 x, within the interval of minus 3 to 4. If we just draw the function, we will see the function is very much unimodal, within minus 3 to 4 and function is having a minimum somewhere in between minus 3 to 4 and there is one restriction is that we want to have the optimal value within 5 percent of exact value. We need to use the Fibonacci method.

As I said the first necessity function must be unimodal ,that is true here the second is that the number of experiments must be specified beforehand. But there is no information as such about the number of experiment. But we can deduce it from the given desire accuracy, the we want to have the accuracy 5 percent accuracy of the exact value, 5 percent of the exact value. That is why if we say L n is the interval after n th experiment, if this if this is my L n, then mid value will be declared as the optimal value I said. Then if you want to have the 5 percent of the exact value, then the half of the n th interval of uncertainty must be less than is equal to L naught into error percentage. That is why we can write it down that, what is my objective?

My objective is to find out the number of experiments from the given, from the level of desire accuracy, length of final interval of uncertainty after n th experiment, divided by length of initial interval of uncertainty. Must be less than is equal to 5 percent. That means, actually we are considering half length of the interval of this is must be is equal to 2 into this. That is why, this must be is equal to 5 by 100. In other way, we can say as L n by 2 must be less than is equal to 1 by 20 L naught or we can say L n must be is equal to less than is equal to L naught by 10. One nice fact of Fibonacci method is that, as I said the measure of efficiency L n by L 0, that is after the n th experiment, if L n is the n th L n is the interval of uncertain, L n is the initial interval of uncertainty, always we will see for any number of n. Whatever number of n we will choose, we will see L n by L 0 always will be is equal to 1 by F n. If we consider n is equal to 5, it would be 1 by F 5. If we consider n is equal to 6, it would be 1 by F 6.

That is why other way, we can say that this measure this reduction ratio can be used for selection of the number of experiments, if it is not given to us. That is why, we can say that L n by L 0 that is equal to 1 by F n must be less than is equal to 1 by 10. What exactly we are getting, F n must be is greater than equal to 10 and we know F 0 is equal to 1, F n is equal to 1 F 2 is equal to 2 ,F 3 is equal to 3, F 4 is equal to 5, F 5 is equal to 8, F 6 is equal to 13. That is why we can conclude here that since F n is greater than equal to 10, that is why we can say n must be greater than equal to 6. This is my conclusion, that is why from here we can say the minimum 6 number of experiments we have to do for getting the desired level of accuracy.

That is the 5 percent of exact value. As we have considered the middle point of the middle point of L n as the optimal solution of the of the given problem. That is why we are concluding from here that the minimum number of n must be 6. If we just increase the number of n as 7, 8 etcetera then we will get better and better result. That is why the interval of uncertainty will be smaller and smaller further. But since, we are doing it manually that is why we will restrict ourselves to value of n as equal to 6. (Refer Slide Time: 30:30)



We will apply in the next the iteration process from step 1. What is my step 1, initial interval of uncertainty is given for us. That is why step 1 would be is equal to L 0 would be equal to from minus 3 to 4 and my n is given we are considering n is equal to 6.

Now, step 2. We have to obtain 2 approximations x 1 and x 2. Now for that thing, as I said in the as I said in the algorithm, we have to generate L 2 star. how to get L 2 ? L 2 star must be is equal to F n minus 2 divided by F n L 0. What is my F n minus 2 here? My n is equal to 6. That is why it must be is equal to F 6 by F 4 by x 6 L 0. What is F 4? F 4 equal to 5. As we have seen F 4 equal to 5, 5 by 13 into L 0, the length. Here length is coming as 7. That is why we will multiple with 7 and this value we will 2.6923. Once we are getting L 2 star, we will generate 2 experiments x 1 and x 2 in the next. How to get x 1?

As I said, if I just consider the interval a to b, that is from minus 3 to 4, then x 1 and x 2 we will generate in such a way that both the experiments will be L 2 star distance apart from both the n's. That is why x 1 must be is equal to minus 3 plus L 2 star, that is

2.6923 and we are getting x 1 is equal to minus 0.3077. And if we consider x 2, we will get 4 minus 2.6923 and that would be is equal to 4 1.3077. That is 1.3077 and function is unimodal in between, that is why our next task is to find out the functional values at x 1 and x 2. What is the functional value at x 1?

Let us see, before to that let me see the functional value at both the ends. If we consider here the functional value with will come as 3 and f x will be 24 at 0.4. At x 1, we will see the functional value is coming, f x 1 is coming as, that is minus not in the positive side. That is in the negative side minus 0.5207 and x 2 value is coming f x 2 value, that is coming 4 3 2, 4 point sorry 4.3255. If we just write down the functional values here, f x 1 would be is equal to, we know the function is x square plus 2 x if we just substitute the x 1 value there, we will get f x 1 that would be minus 5207 and f x 2 is coming as 4.3255. What we see that, f x 1 is lesser than f x 2.

Since the function is unimodel, the maximum I am sorry, this is the minimization of the if function. The minimum cannot live within this because the functional value function is coming like these only. That is why minimum will be in the within the interval from minus 3 to x 2. That is why we are discarding. This is the conclusion that discard x 2, 4 and obtain the new interval of uncertainty, obtain L 2 that is the new interval uncertainty from minus 3 to x 2.

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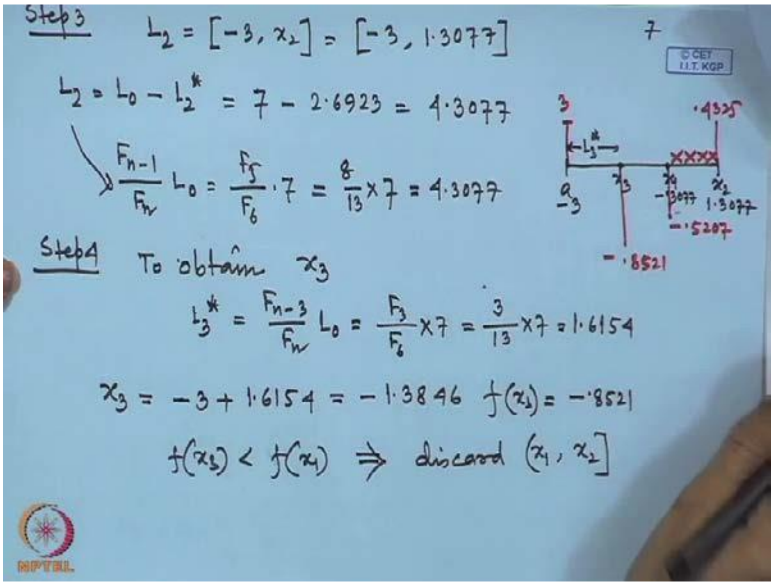

Coming to the next, step 3.We will get L 2 as we have consider L 2 is equal to minus 3 to x 2. That is means minus 3 to 1.3077. If we consider, if we just apply the formula we have learnt, just now learnt we will get the same value for L 2. How?

As we know, L 2 is equal to L naught minus L 2 star. what is L naught, that the length of L naught would was 7. Whenever I am writing L naught L 2 star within the equation it means that, this is the length of the given interval. And L naught is 7 and what about L 2 star. Just now we got L 2 star is equal to 2.6923 and that is coming as 4.3077. If we just see the interval size here, it is coming 4.3077 and again there is another nice fact is that, we can get L 2 in this way as well. As we got that L 2 is equal to F n minus 1 by F n L 0. What is F n minus 1? That is my F 6, that is my F 5. L 0 is 7 this is another formula for L 2. This is coming as 8 by 13 into 7 is equal to 4.3077. That is the nice fact, this is the beauty of the Fibonacci method.

In if we apply this, if we just consider this one, its coming the same value here also the same value because we are considering here the Fibonacci numbers. But the basic idea is that now in the next we will consider the interval from a to x 2. What is my a? That is minus 3, my x 2 is 1.3077. In between, there is a point we are having that is x 1. My x 1 is equal to minus 0.3077, that is my x 1. Then what is my next step? Next step would be is equal to obtain the next approximation for the optimal solution, that is x 3. For getting x 3 again we need to generate L 3 star. As we have done before, L 3 star is equal to F n minus 3 divided by F n L 0. What is F n minus 3? This is F 6, this is f 3. L 0 is again 7

here and F 6 is equal to 13 and this F 3 is equal to 3. That is why, 3 by 13 into 7 this value is coming 1.6154.

Once we are getting so, very nicely we can select x 3 in such a way that, the current 2 experiments, that is the experiment 1 is x 1 and another one is x 3. That would be L 3

star distance apart from both the ends of the given interval. And if we just see the fact x 1

is L 3 star distance apart because if I just consider the length of the interval from x 1 to x

2, always it would be is equal to 1.6154. That is why we have to select x 3 here. That where x 3 would be L 3 star distance apart from one end. And in this way, we will get x 3 is equal to my a plus L 3 star. That is, minus 3 plus 1.6154 and it will come as minus 1.3846 and if we just find out the functional value at this point, we will get the functional value as minus 0.8521.

Now, let us see what is happening in the figure and if we see again at point minus 3 the value is coming 3. At point x 2 the value is, we calculated the value is 0.4325. At point x 1, the value is coming minus 0.5207. At point x 3, the value is coming minus 0.8521. What we see again? Again we see that, f x 3 is lesser than f x 1. That is why what is my conclusion? Discard this interval from optimum cannot lie within this because minimum should lie within this interval. That is why discard the interval x 1 to x 2 and we getting new interval of uncertainty L 3 as minus 3 to minus 0.3077.

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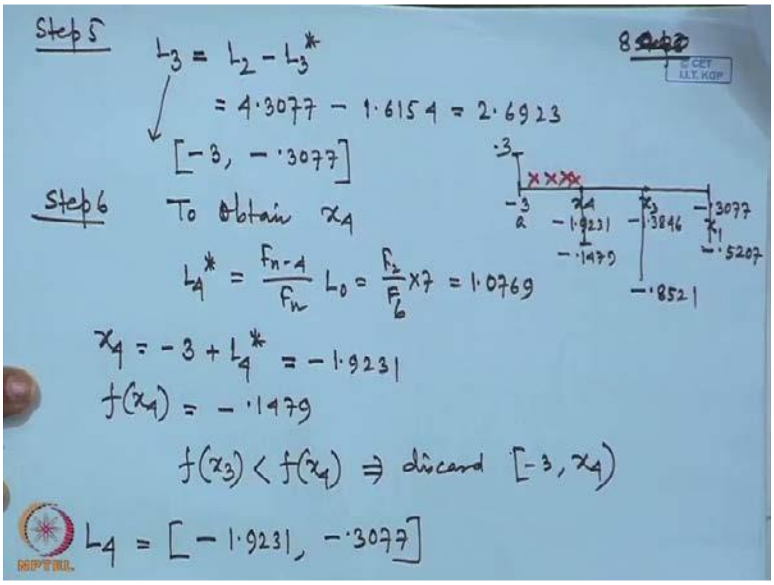

That is why let us go to the next step, step 5. I am sorry, this is not the step 5. This is page numbers 8 and we are going to step 5 and we will get L 3 equal to L 2 minus L 3 star. My L 2 was the length of L 2 rather 4.3077 and L 3 star, just now we have calculated 1.6154. That is why the length is coming 1.6923.

If this is so, then we are reaching to the next interval. This interval, L 3 interval will be from minus 3 to minus 3077. What is the next task? Again the next task would be to find out the next approximation. We have to reach up to sixth experiment because we have consider n is equal to 6. That is why we are moving to the next. We will obtain x 4. Let me draw the figure again. We are having minus 3 in 1 end, minus 0.3077 at x 1 in the other end. This is my a, in between we are having x 3 and this value is, x 3 is equal to just now we got that is minus 1.3846.

Now, to obtain x 4 again, we need to find out L 4 star. That would be is equal to F n minus 4 divided by F n L 0. Let me just calculate the entire thing very quickly. That would be F 2 into F 6 into 7, that will come as 1.0769. If this is so, again the same thing is that, both the current 2 experiments must be L 4 star distance apart from both the ends . That is why we will consider the next, that is x 4 as minus 3 plus L 4 star and it would be is equal to minus 1.9231 and here must be minus 1.9231. Again we will see the functional value at x 4. We will see the function, we see the functional value is coming as minus 1 point 0.1479. What we see the figure here again, here it is 3, here it is minus 0.5207.

Here it is minus 0.8521 and here at x 4 the value is coming as minus 0.1479. What we see again? We see that f x 3 is lesser than f x 4, then what is the conclusion? Discard the interval from minus 3 to x 4. Minus 3 to x 4 means we are discarding this interval in the next, that is why the new interval of uncertainty. That is my L 4 will become as from x 4, that is minus 1.9231 to minus 3077. We have to repeat the process again let us go to the next.

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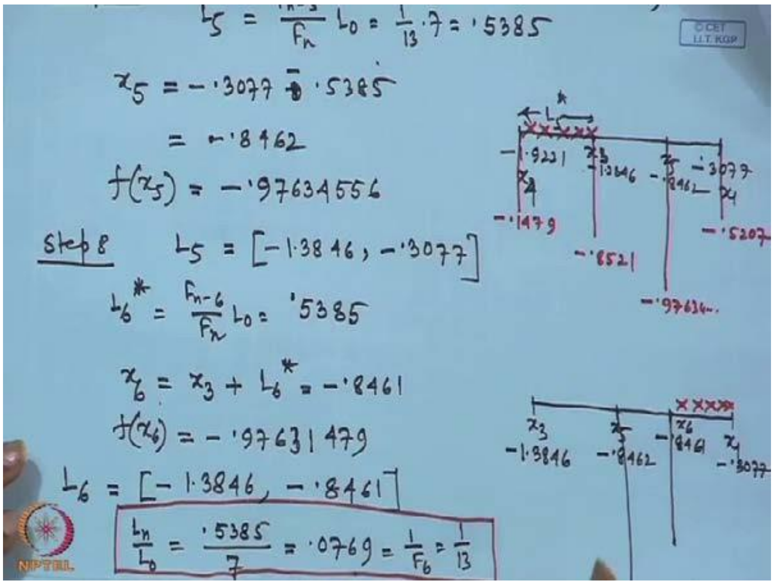

We have to have more number of experiments, then only we will get better result. We cannot start with n equal to 2 or 3. This is a process, if you just automate the process, the process is very easy. But if we just do it manually, it takes time. L 5 star, L 5 star again would be equal to F n 5 F n minus 5 by F n L 0, that would be is equal to 1 by 13 into 7 that is coming as 5385.

Therefore, we will again considered the interval from x 3 to x 1. That was minus 1.9231 to minus 0.3077, that is my x 1, that is my x 3 here. In between we are having x 4. Now that is my x 4. We are having x 3 in between. x 3 is coming somewhere. Now, we will calculate x 5. Again the same logic, current two experiments, that is x 3 and x 5 must be L 3 L 5 star distance apart from both the ends. If we see that it will be always this is L 5 star. That is why x 5 must be here somewhere, that is why we will consider x 5 is equal to minus 0.3077 plus L 5 star, that is 0.5385. If we consider, we will get x 5 is equal to minus 0.8462. This 1 x 3 was minus 1.3846 and we are having x 5 as minus 0.8462. This is minus.

And we will calculate the functional value at x 5 and this value is coming 0.97. We are taking a large number here, with more decimal points because we will see why we are considering in the next. Because once we will to we will go to x 6, that will be very close to x 5 in the next we will see and that is why, we will see that in the next case functional value will differ with a small amount. That is why we are we are considering a more decimal points in this case. And here we are having x 4, this is my x 4. Now let us see the functional values at every point. At x 4, it was minus 0.1479 and x 3, the value was minus

0.8521. At x 5 the functional value is more, that is 0.976344556 and at point x 1 the functional value is minus 0.5207.

Again the same conclusion, since the function is an unimodel x f x 5 is lesser than f x 3. Therefore, optimal cannot lie in this area. That is why, we will discard this interval and we will get the next interval L 5 as minus 1.3846 to minus 0.3077. Alright and in between how many points? We are having from the previous one, we are having x 3, we are having x 5. Let me write down the values here, 1.3846. My x 5 was minus 8462, my x 1 was minus 3077. We need to get x 6 because that is my last approximation. Let us see what is happening in the next. We will consider L 6 star, that would be F n minus 6

by F n L. That is again F 0 by F 6 by into L 0. That is why, its again 1 by 13 into 7, that is why the L 6 star value is same as the previous value. Always we will see the last, L n star would be is equal to L n minus 1 star always, if the n is number of experiments.

If this is so, then we will say the same thing, that the current 2 experiments would L 6 stars distance apart from both the ends. That is why x 6 would be is equal to x 3 plus L 6

star and we will get this value as minus 8461. And what is the functional value at x 6? We will see the functional value is coming minus 0.9761. Just see with the f x 5, 61631 sorry 631479. That is why, what we see we are getting x 6 here, that is minus. This is my x 6, 8461 and we are having lesser functional value, higher functional value than x 5. That is why, what is the conclusion in the next? Since this is high value, considering the unimodality condition, again we will discard the this part of the interval and we will declare that the final interval of uncertainty L 6 would be is equal to from minus 1.3846 to minus 0.8461.

This is the and the middle point of this interval will be declared as the optimal solution, for this interval. Now there are few things to be noted here. The first thing is that, always we will see that, x n minus 1 is almost same as x n and only the difference will be there in the decimal, higher in the higher decimal points. This is almost same with x. x n is same as x n minus 1. And another nice fact is that, always we will see that L n by L 0, that is my reduction ratio that value here, if we see the L n distance, this distance will be is equal to 5385 and L 0 is my 7. This is equal to, we will see 0.0769 and that would be is equal to 1 by F n.

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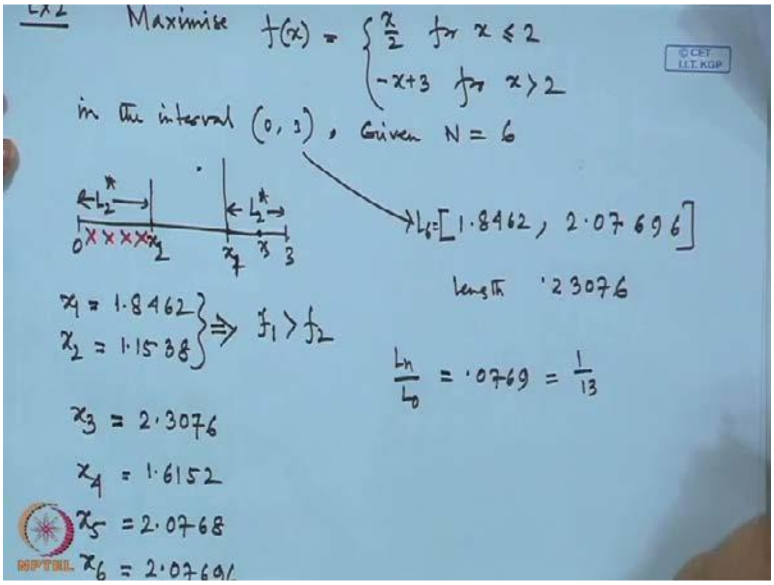

What is my n here? n equal to 6. That is why it would be 1 by F 6, rather this will be 1 by 13. That is the nice fact of the Fibonacci method. Always we will get this one. For whatever n value we will consider, is irrespective the value of n, we will get the same result here. Let us move to the next problem and this is our maximization problem.

Let me do the problem very quickly because I will just write down the steps. Without going into detail about the calculations, my problem is that maximize f x, x by 2 for x lesser is equal to 2. And this is minus x plus 3, for x greater than 2. We must see the pattern of the function. Pattern of the function is very small here. I am sorry, the pattern of the function is that function is not really continuous and we need to maximize within the interval 0 to 3.

And using the Fibonacci method and another thing is given here, that number of experiment has been specified n is equal to 6, alright. For starting the process, again we will do the same process. We will consider a and b, that is 0 to 3. We will consider L 2 star and we will find out x 1 and x 2. L 2 star distance apart from both the ends. And we will see x 1 will come as 1.8462, x 2 will become x 2 will be 1.1538. We will find out the functional values at both the points here. And we will see the functional value will come, just you calculate. We will see, this will give you the value - f 1 is higher value than F 2. Since the problem is the maximization problem, that is why the discard process is the reverse to the minimization problem.

Here, we will not discard the higher value. We will discard the lower value. That is why the this would be this one, this is the higher value and we will discard this interval in the first case. And we will move to the next. We will calculate L 3 star and once we will calculate L 3 star, we can generate x 3. x 3 would be is equal to 2.3076 and this is my interval and we will place x 3 in such a way that, it would be L 3 star distance apart from both the ends. And we will see that x 3 will be, I am sorry this is x 2 and this is coming x 1. And if we, this is so, we will consider x 3 here and we will get the functional value and functional value will, again we will see that f x 3 value is lesser than f x 1 value. In that way, we can complete the whole process.

Let me write down the approximations only. x 1, x 2, x 3, x 4 is coming as 1.6152. x 5 is coming as 2.0768 and x 6 ultimately will come as 2.07696. And if we just do the, apply the same process, this initial interval of uncertainty will reduce at the end to 1.x 4 62 to

2.07696. That would be L 6, rather because we are doing the 6 number of experiments, all together. Here the length of L 6 is coming.

Again 0.23076 and here also we can just see the same fact, that L n by L 0 that is the L 6 by L 0 in that case. This divided by 3 is equal to we will get 0769. The same value 1 by 13, 1 by F 6. The entire calculation, anybody can do just like the previous problem, just only thing just I wanted to mention here that, for the maximization problem, if we just that unimodality property, instead of considering the lower value of function, we will considered the higher value of function in the selection.

And that is why the discard, the elimination of the interval will be according to that and in that way we will reach to the final interval of uncertainty. That would be much more smaller length and that can be declared as the final interval of uncertainty and middle value of that final interval of uncertainty will be the optimal solution. Fibonacci method is very efficient method for getting optimal solution, for non-linear programming problem even for discontinuous function. That is all for today.

Thank you.