# Graph Coloring

## Basic Definitions

1. **Graph coloring**

   o Is a simple way of labelling graph components such as vertices, edges, and regions under some constraints, by coloring them.

   o In a graph, no two adjacent vertices, adjacent edges, or adjacent regions are colored with same color

2. **Chromatic Number:**

   o The minimum number of colors needed to color a graph G is called its chromatic number.

   o Example 1

   ▪ The following can be colored with minimum 3 colors, therefore its **chromatic number is 3**
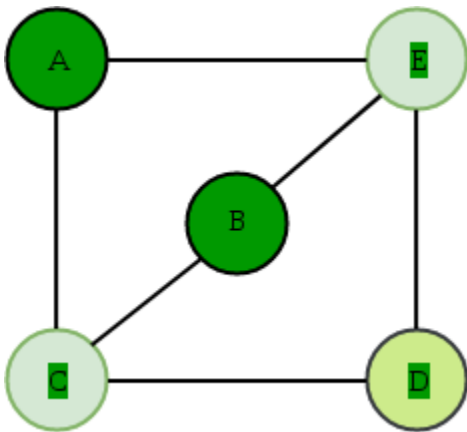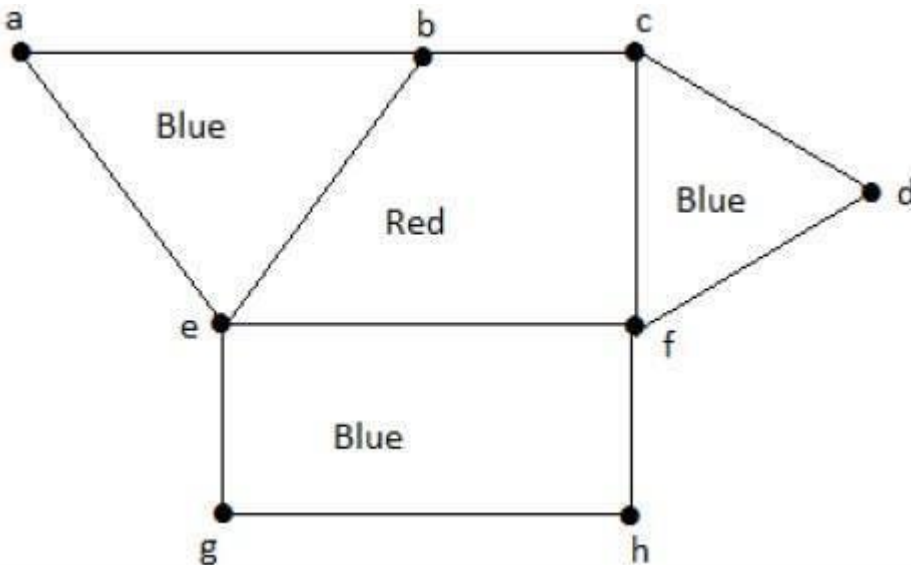


   ▪ Example 2

   • In the following graph the regions 'aeb' and 'befc' are adjacent, as there is a common edge 'be' between those two regions. Similarly, the other regions are also colored based on the adjacency.
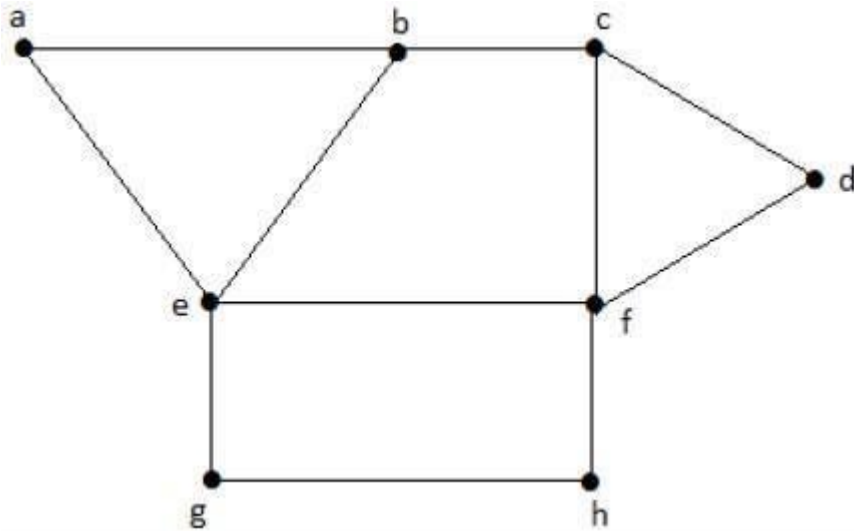
This graph can be colored as follows:



**Here, the chromatic number of graph = 2**

3. **The Four Color Theorem:** The chromatic number of a planar graph is no greater than four

# Applications of Graph Coloring

1. Data mining
2. Networking
3. Resource allocation
4. Processes scheduling
5. Scheduling exams
   - So that there is no clash of exams for any student
6. Creating time tables
   - So that there is no clash of classes for any student or faculty member
7. Map coloring
   - Each adjacent region is to be colored with a different color
   - Geographical maps of countries or states where no two adjacent cities can be assigned same color.
   - Four colors are sufficient to color any map.
8. Mobile Radio Frequency Assignment:
   - When frequencies are assigned to towers, frequencies assigned to all towers at the same location must be different.
   - How to assign frequencies with this constraint?
   - What is the minimum number of frequencies needed?
   - This problem is also an instance of graph coloring problem where every tower represents a vertex and an edge between two towers represents that they are in range of each other.
9. Sudoku:
   - Sudoku is also a variation of Graph coloring problem where every cell represents a vertex.
   - There is an edge between two vertices if they are in same row or same column or same block.
10. Register Allocation:
    - In compiler optimization, register allocation is the process of assigning a large number of target program variables onto a small number of CPU registers.
    - This problem is also a graph coloring problem.

   And many more…