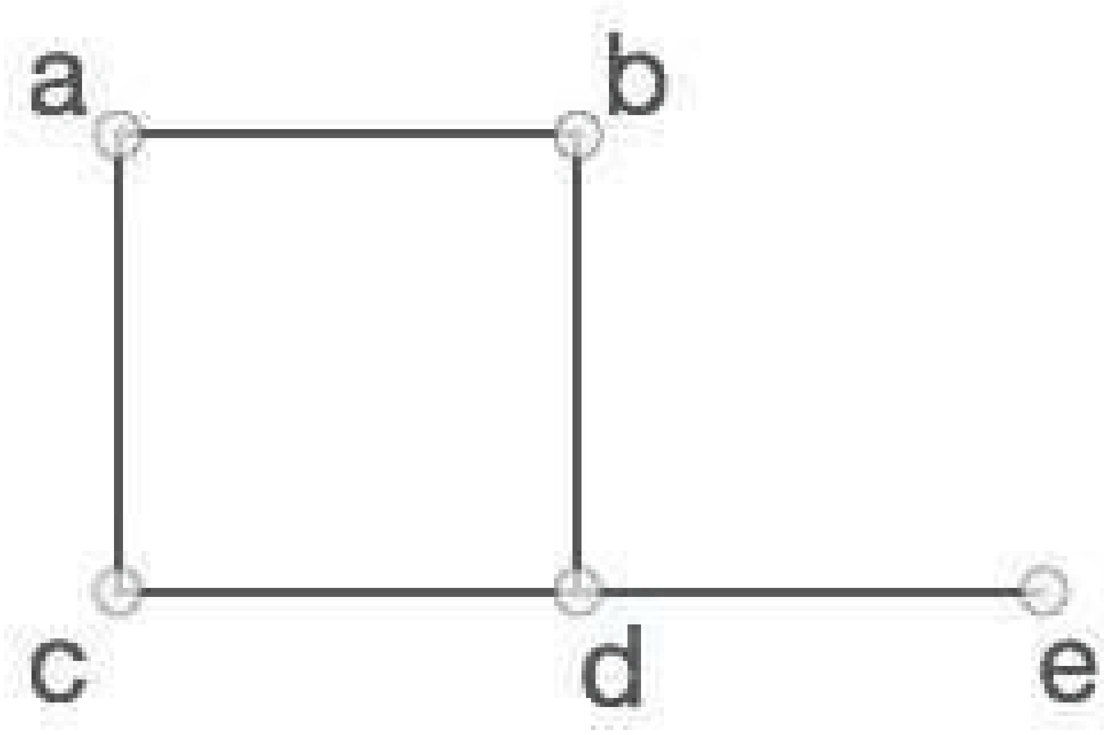
Graphs:

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as **vertices,** and the links that connect the vertices are called **edges**.

Formally, a graph is a pair of sets **(V, E),** where **V** is the set of vertices and **E** is the set of edges, connecting the pairs of vertices. Take a look at the following graph −



In the above graph,

V = {a, b, c, d, e}

E = {ab, ac, bd, cd, de}

Graph Data Structure:

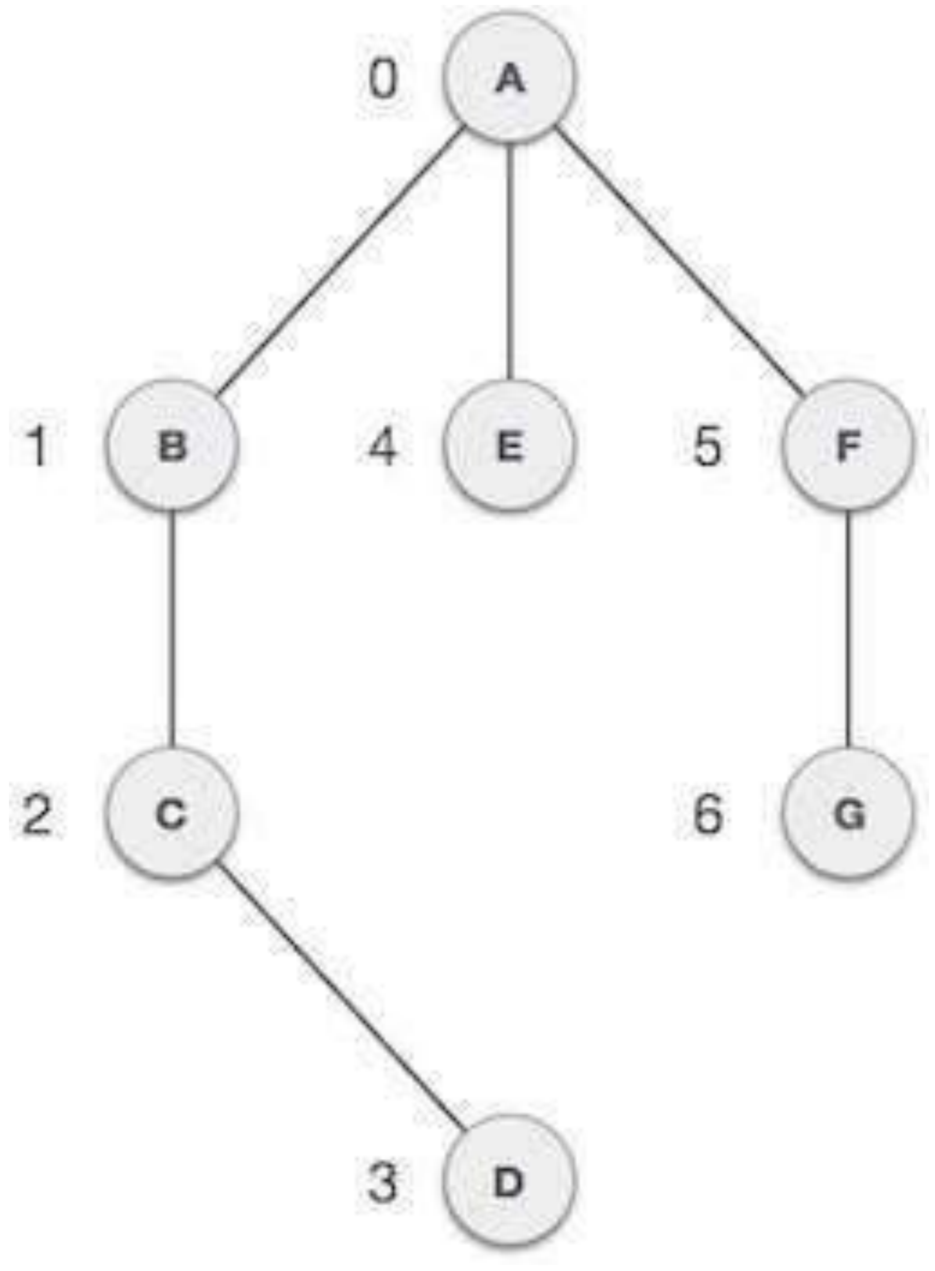
Mathematical graphs can be represented in data structure. We can represent a graph using an array of vertices and a two-dimensional array of edges. Before we proceed further, let's familiarize ourselves with some important terms −

- **Vertex** − Each node of the graph is represented as a vertex. In the following example, the labeled circle represents vertices. Thus, A to G are vertices. We can represent them using an array as shown in the following image. Here A can be identified by index 0. B can be identified using index 1 and so on.

- **Edge** − Edge represents a path between two vertices or a line between two vertices. In the following example, the lines from A to B, B to C, and so on represents edges. We can use a two-dimensional array to represent an array as shown in the following image. Here AB can be represented as 1 at row 0, column 1, BC as 1 at row 1, column 2 and so on, keeping other combinations as 0.

- **Adjacency** − Two node or vertices are adjacent if they are connected to each other through an edge. In the following example, B is adjacent to A, C is adjacent to B, and so on.

- **Path** − Path represents a sequence of edges between the two vertices. In the following example, ABCD represents a path from A to D.



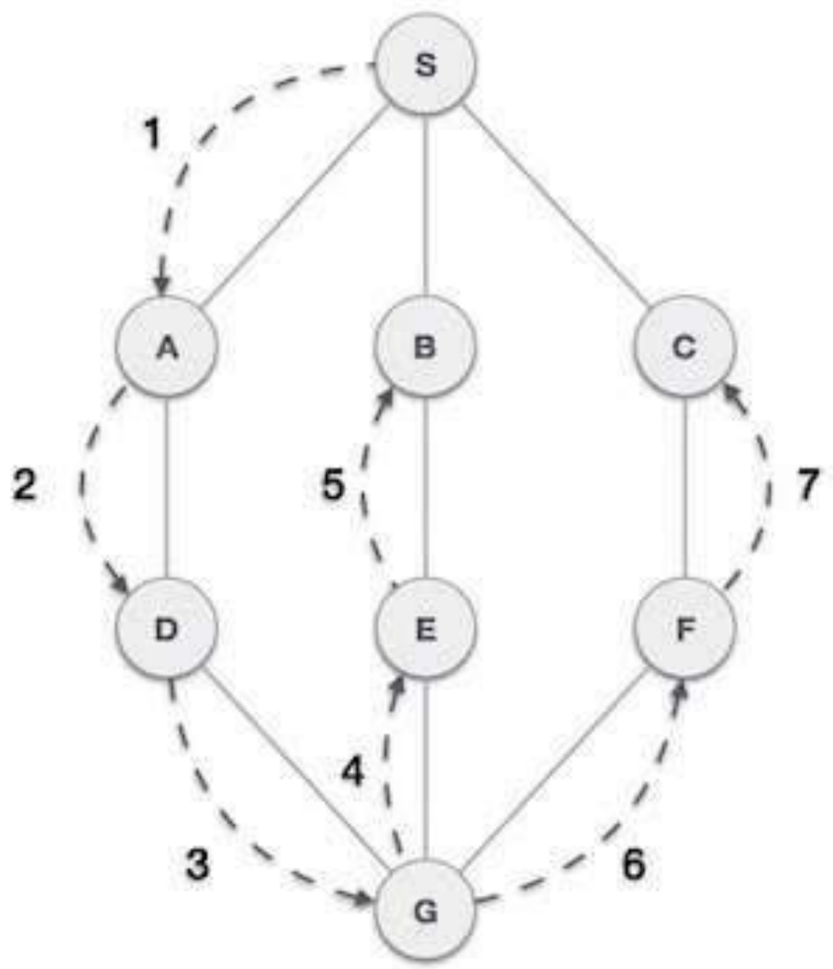
Basic Operations:

Following are basic primary operations of a Graph which are following.

- **Add Vertex** − Adds a vertex to the graph.
- **Add Edge** − Adds an edge between the two vertices of the graph.
- **Display Vertex** − Displays a vertex of the graph.

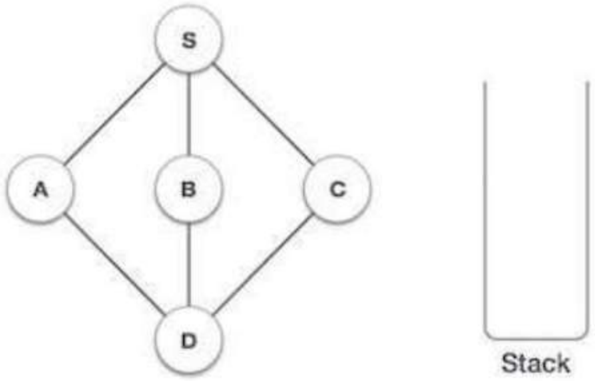
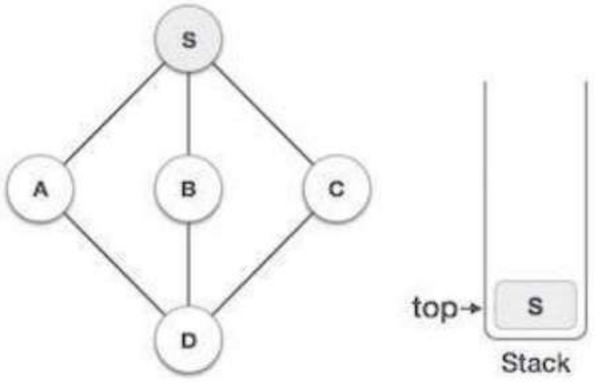
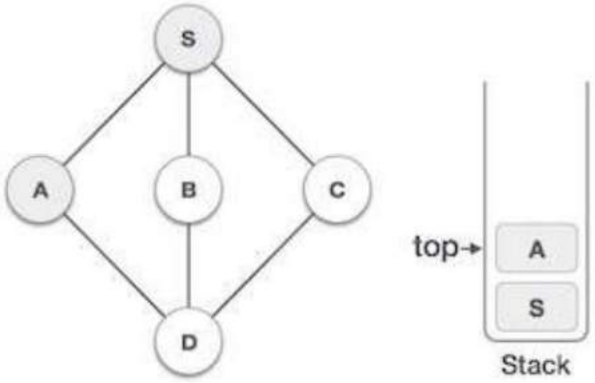
Depth First Search (DFS):

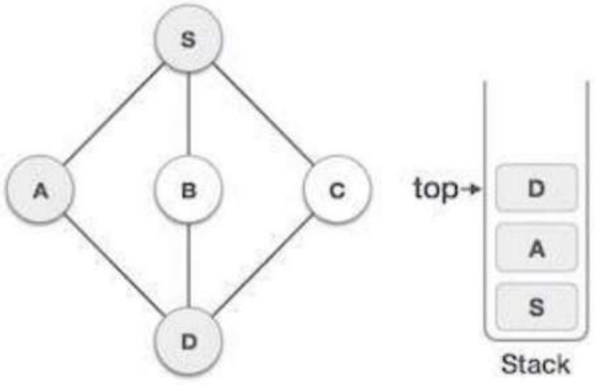
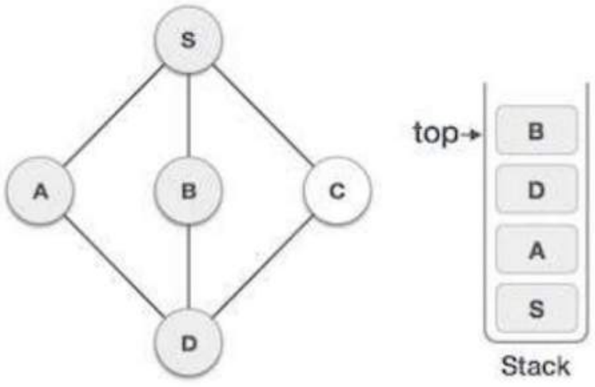
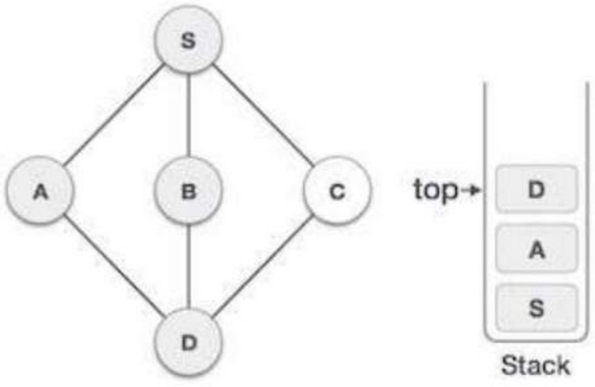
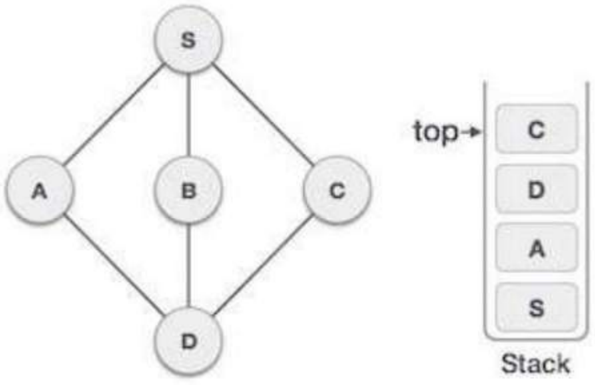
Depth First Search (DFS) algorithm traverses a graph in a depthward motion and uses a stack to remember to get the next vertex to start a search, when a dead end occurs in any iteration.



As in the example given above, DFS algorithm traverses from A to B to C to D first then to E, then to F and lastly to G. It employs the following rules.

- **Rule 1** – Visit the adjacent unvisited vertex. Mark it as visited. Display it. Push it in a stack.
- **Rule 2** – If no adjacent vertex is found, pop up a vertex from the stack. (It will pop up all the vertices from the stack, which do not have adjacent vertices.)
- **Rule 3** – Repeat Rule 1 and Rule 2 until the stack is empty.

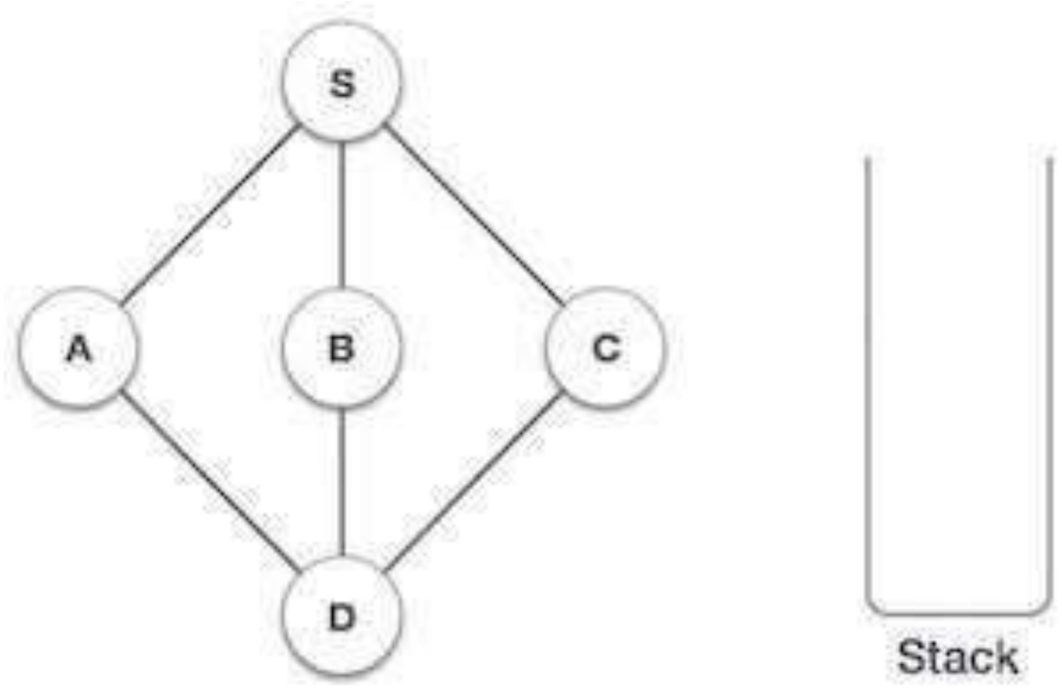
Steps	Traversal	Description
1.		Initialize the stack.
2.		Mark **S** as visited and put it onto the stack. Explore any unvisited adjacent node from **S**. We have three nodes and we can pick any of them. For this example, we shall take the node in an alphabetical order.
3.		Mark **A** as visited and put it onto the stack. Explore any unvisited adjacent node from A. Both **S** and **D** are adjacent to **A** but we are concerned for unvisited nodes only.

4.		Visit **D** and mark it as visited and put onto the stack. Here, we have **B** and **C** nodes, which are adjacent to **D** and both are unvisited. However, we shall again choose in an alphabetical order.
5.		We choose **B**, mark it as visited and put onto the stack. Here **B** does not have any unvisited adjacent node. So, we pop **B** from the stack.
6.		We check the stack top for return to the previous node and check if it has any unvisited nodes. Here, we find **D** to be on the top of the stack.
7.		Only unvisited adjacent node is from **D** is **C** now. So we visit **C**, mark it as visited and put it onto the stack.

As **C** does not have any unvisited adjacent node so we keep popping the stack until we find a node that has an unvisited adjacent node. In this case, there's none and we keep popping until the stack is empty.

Depth First Traversal in C:

We shall not see the implementation of Depth First Traversal (or Depth First Search) in C programming language. For our reference purpose, we shall follow our example and take this as our graph model −



Implementation in C:

```
#include <stdio.h>
#include <stdlib.h>
#include <stdbool.h>

#define MAX 5

struct Vertex {
   char label;
   bool visited;
};
```

```
//stack variables

int stack[MAX];
int top = -1;



//graph variables

//array of vertices
struct Vertex* lstVertices[MAX];

//adjacency matrix
int adjMatrix[MAX][MAX];

//vertex count
int vertexCount = 0;

//stack functions

void push(int item) {
   stack[++top] = item;
}

int pop() {
   return stack[top--];
}

int peek() {
   return stack[top];
}

bool isStackEmpty() {
   return top == -1;
}
```

```
//graph functions

//add vertex to the vertex list
void addVertex(char label) {
   struct Vertex* vertex = (struct Vertex*) malloc(sizeof(struct
   Vertex)); vertex->label = label;
   vertex->visited = false;
   lstVertices[vertexCount++] = vertex;
}

//add edge to edge array
void addEdge(int start,int end) {
   adjMatrix[start][end] = 1;
   adjMatrix[end][start] = 1;
}

//display the vertex
void displayVertex(int vertexIndex) {
   printf("%c ",lstVertices[vertexIndex]->label);
}

//get the adjacent unvisited vertex
int getAdjUnvisitedVertex(int vertexIndex)
   { int i;

   for(i = 0; i<vertexCount; i++) {
      if(adjMatrix[vertexIndex][i] == 1 && lstVertices[i]->visited == false)
         { return i;
      }
   }

   return -1;
}
```

```
void depthFirstSearch() {
   int i;

   //mark first node as visited
   lstVertices[0]->visited = true;

   //display the vertex
   displayVertex(0);

   //push vertex index in
   stack push(0);

   while(!isStackEmpty()) {
      //get the unvisited vertex of vertex which is at top of the
      stack int unvisitedVertex = getAdjUnvisitedVertex(peek());

      //no adjacent vertex found
      if(unvisitedVertex == -1) {
         pop();
      }else {
         lstVertices[unvisitedVertex]->visited =
         true; displayVertex(unvisitedVertex);
         push(unvisitedVertex);
      }
   }

   //stack is empty, search is complete, reset the visited flag
   for(i = 0;i < vertexCount;i++) {
      lstVertices[i]->visited = false;
   }
}
```

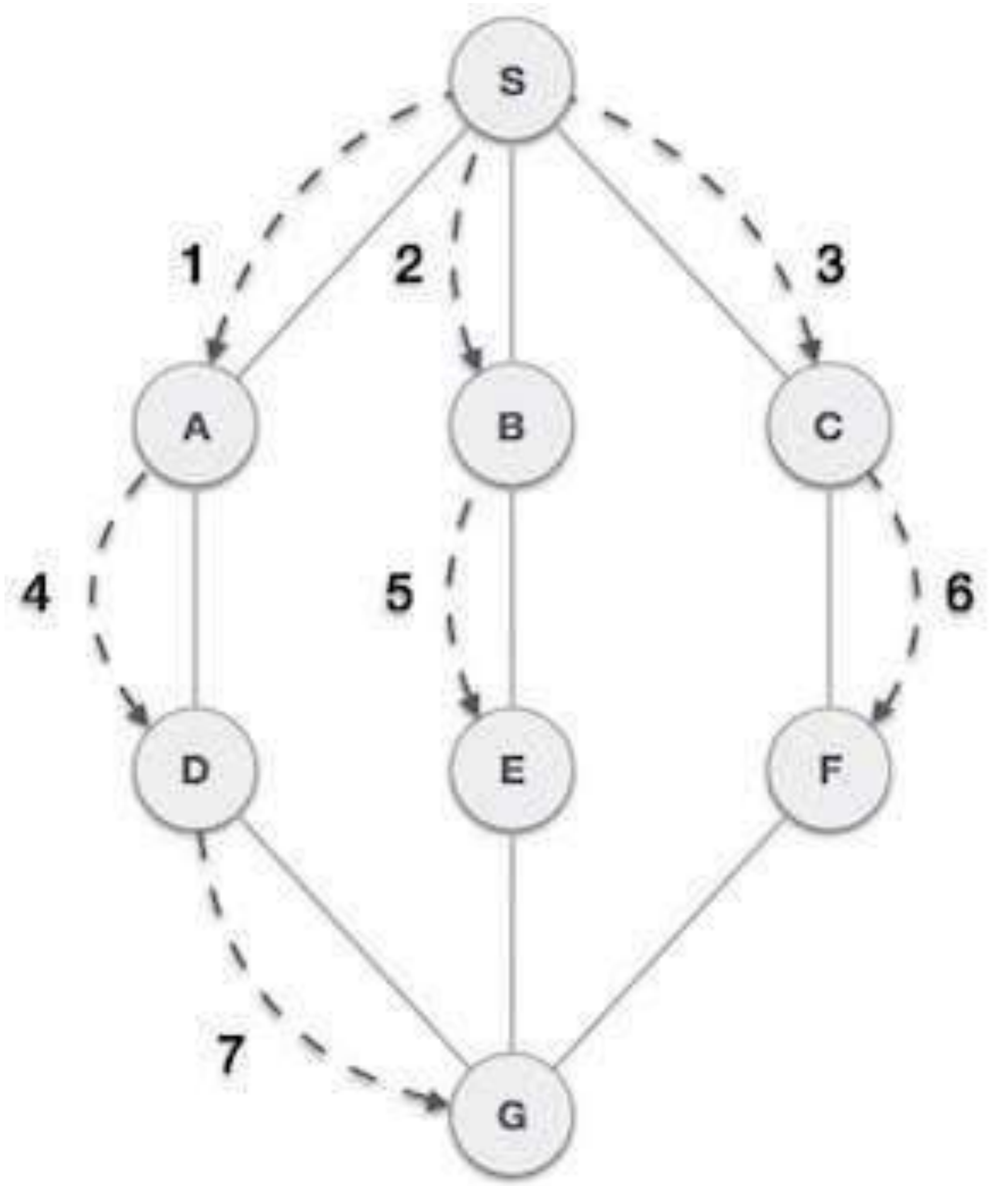
```
int main() {
   int i, j;

   for(i = 0; i<MAX; i++) // set adjacency {
      for(j = 0; j<MAX; j++) // matrix to 0
         adjMatrix[i][j] = 0;
   }

   addVertex('S');   // 0
   addVertex('A');   // 1
   addVertex('B');   // 2
   addVertex('C');   // 3
   addVertex('D');   // 4

   addEdge(0, 1);    // S - A
   addEdge(0, 2);    // S - B
   addEdge(0, 3);    // S - C
   addEdge(1, 4);    // A - D
   addEdge(2, 4);    // B - D
   addEdge(3, 4);    // C - D

   printf("Depth First Search: ");

   depthFirstSearch();

   return 0;
}
```

If we compile and run the above program, it will produce the following result −

```
Depth First Search: S A D B C
```

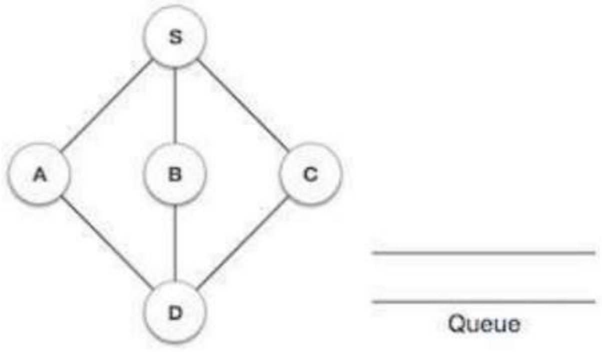
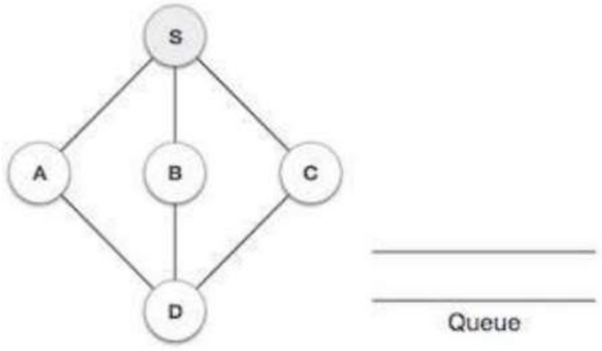
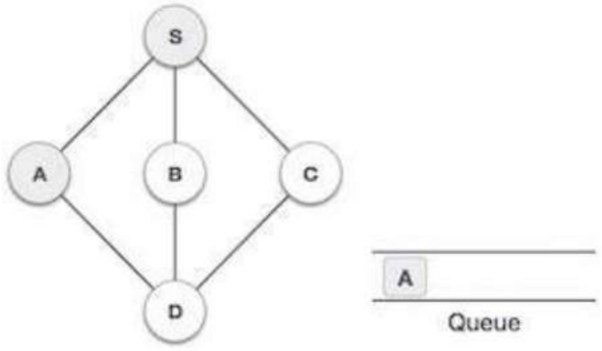
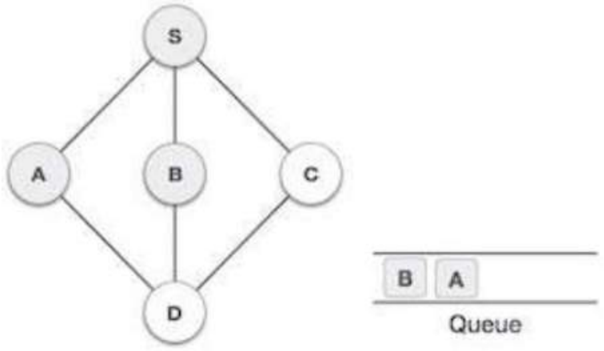
Breadth First Search (BFS):

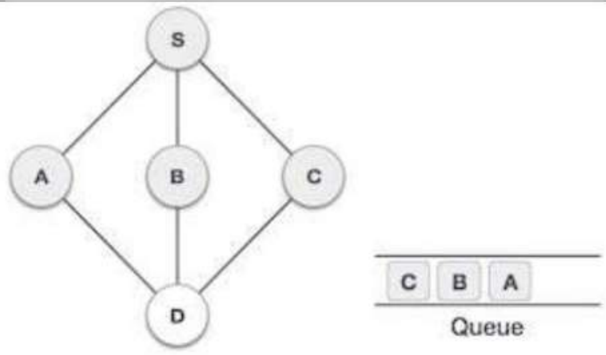
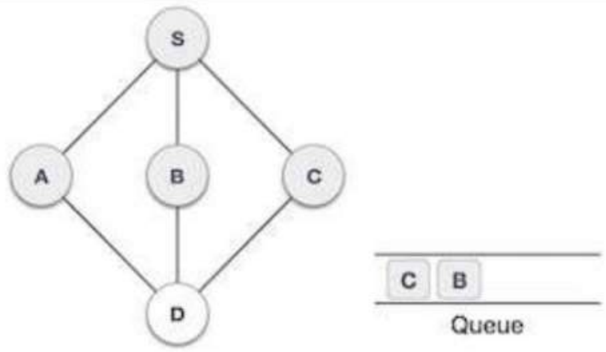
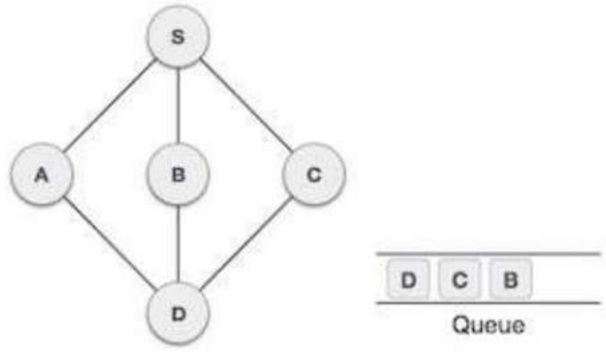
Breadth First Search (BFS) algorithm traverses a graph in a breadthward motion and uses a queue to remember to get the next vertex to start a search, when a dead end occurs in any iteration.



As in the example given above, BFS algorithm traverses from A to B to E to F first then to C and G lastly to D. It employs the following rules.

- **Rule 1** − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Insert it in a queue.
- **Rule 2** − If no adjacent vertex is found, remove the first vertex from the queue.
- **Rule 3** − Repeat Rule 1 and Rule 2 until the queue is empty.

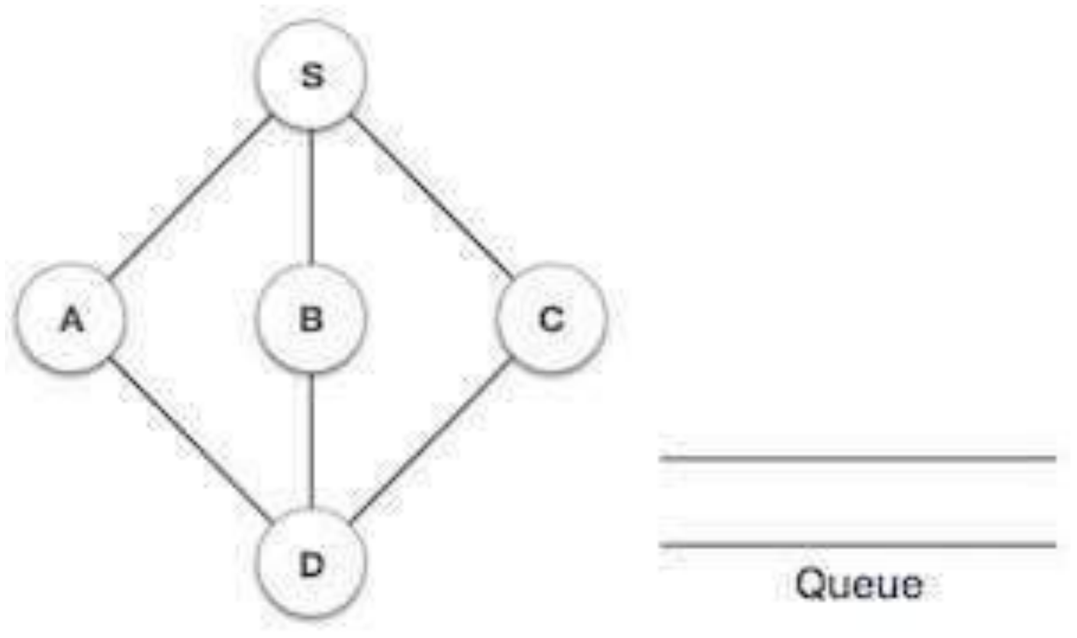
Steps	Traversal	Description
1.		Initialize the queue.
2.		We start from visiting **S** (starting node), and mark it as visited.
3.		We then see an unvisited adjacent node from **S.** In this example, we have three nodes but alphabetically we choose **A,** mark it as visited and enqueue it.
4.		Next, the unvisited adjacent node from **S** is **B**. We mark it as visited and enqueue it.

5.		Next, the unvisited adjacent node from **S** is **C**. Mark it as visited and enqueue it.
6.		Now, **S** is left with no unvisited adjacent nodes. So, we dequeue and find **A**.
7.		From **A** we have **D** as unvisited adjacent node. We mark it as visited and enqueue it.

At this stage, we are left with no unmarked (unvisited) nodes. But as per the algorithm we keep on dequeuing in order to get all unvisited nodes. When the queue gets emptied, the program is over.

Breadth First Traversal in C:

We shall not see the implementation of Breadth First Traversal (or Breadth First Search) in C programming language. For our reference purpose, we shall follow our example and take this as our graph model −



Implementation in C

```
#include <stdio.h>
#include <stdlib.h>
#include <stdbool.h>

#define MAX 5

struct Vertex {
   char label;
   bool visited;
};

//queue variables

int queue[MAX];
int rear = -1;
int front = 0;
int queueItemCount = 0;

//graph variables
```

```
//array of vertices
struct Vertex* lstVertices[MAX];

//adjacency matrix
int adjMatrix[MAX][MAX];

//vertex count
int vertexCount = 0;


//queue functions

void insert(int data) {
   queue[++rear] = data;
   queueItemCount++;
}

int removeData() {
   queueItemCount--;
   return queue[front++];
}

bool isQueueEmpty() {
   return queueItemCount == 0;
}

//graph functions

//add vertex to the vertex list
void addVertex(char label) {
   struct Vertex* vertex = (struct Vertex*) malloc(sizeof(struct
   Vertex)); vertex->label = label;
   vertex->visited = false;
   lstVertices[vertexCount++] = vertex;
}
```

```
//add edge to edge array
void addEdge(int start,int end) {
   adjMatrix[start][end] = 1;
   adjMatrix[end][start] = 1;
}

//display the vertex
void displayVertex(int vertexIndex) {
   printf("%c ",lstVertices[vertexIndex]->label);
}

//get the adjacent unvisited vertex
int getAdjUnvisitedVertex(int vertexIndex)
   { int i;

   for(i = 0; i<vertexCount; i++) {
      if(adjMatrix[vertexIndex][i] == 1 && lstVertices[i]->visited ==
         false) return i;
   }

   return -1;
}

void breadthFirstSearch() {
   int i;

   //mark first node as visited
   lstVertices[0]->visited = true;

   //display the vertex
   displayVertex(0);
   //insert vertex index in
   queue insert(0);
   int unvisitedVertex;

   while(!isQueueEmpty()) {
      //get the unvisited vertex of vertex which is at front of the
      queue int tempVertex = removeData();

      //no adjacent vertex found
      while((unvisitedVertex = getAdjUnvisitedVertex(tempVertex)) != -1)
         { lstVertices[unvisitedVertex]->visited = true;
         displayVertex(unvisitedVertex);
         insert(unvisitedVertex);
      }

   }

   //queue is empty, search is complete, reset the visited
   flag for(i = 0;i<vertexCount;i++) {
      lstVertices[i]->visited = false;
   }
}

int main() {
   int i, j;

   for(i = 0; i<MAX; i++) // set adjacency {
      for(j = 0; j<MAX; j++) // matrix to 0
         adjMatrix[i][j] = 0;
   }

   addVertex('S');  //  0
   addVertex('A');  //  1
   addVertex('B');  //  2
   addVertex('C');  //  3
   addVertex('D'); // 4
```

```
    addEdge(0, 1);    // S - A
    addEdge(0, 2);    // S - B
    addEdge(0, 3);    // S - C
    addEdge(1, 4);    // A - D
    addEdge(2, 4);    // B - D
    addEdge(3, 4);    // C - D


    printf("\nBreadth First Search: ");

    breadthFirstSearch();

    return 0;
 }
```

If we compile and run the above program, it will produce the following result –

```
Breadth First Search: S A B C D
```

References:

1. “Data Structures & Algorithm” , Tutorials Point India (https://www.tutorialspoint.com)